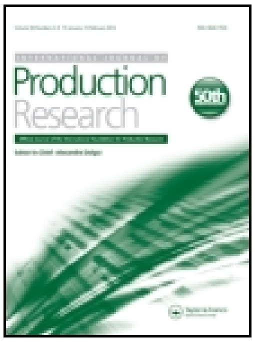

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Contingent sourcing under supply disruption and competition

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With the increasing awareness of the serious consequences of supply disruption risk, firms adopt various kinds of strategies to mitigate it. We consider a supply chain in which two suppliers sell components to two competing manufacturers producing and selling substitutable products. Supplier U is unreliable and cheap, while Supplier R is reliable and expensive. Firm C uses a contingent dual-sourcing strategy and Firm S uses a single-sourcing strategy. We study the implications of the contingent sourcing strategy under competition and in the presence of a possible supply disruption. The time of the occurrence of the supply disruption is uncertain and exogenous, but the procurement time of components is in the control of the firms. We show that supply disruption and procurement times jointly impact the firms' buying decisions. We characterise the firms' optimal order quantities and their expected profits under different cases. Subsequently, through numerical computations, we obtain additional managerial insights. Finally, as extensions, we study the impact endogenizing equilibrium sourcing strategies of asymmetric and symmetric firms, and of capacity reservation by Firm C with Supplier R to mitigate disruption.

Keywords: supply chain management; game theory; risk management

1. Introduction

Supply chain disruption risk is becoming an increasingly important topic of study in operations management. Disruptions in supply chains make suppliers unable to fulfil the product quantities ordered by manufacturers/buyers. Failure to meet demand can be caused by bottlenecks in production or supply processes and natural disasters such as power outages, terrorist attacks and natural hazards. Supply chain disruptions may cause suppliers to default in meeting the manufacturer's orders. Modern business operations such as outsourcing and external procurement are becoming increasingly common, but they tend to make supply chains highly interdependent. With such dependence, a default on the part of an upstream supplier leads to supply disruptions downstream. Therefore, one of the biggest challenges faced by supply chain managers in today's globalised and highly uncertain business environment is to proactively and efficiently prepare for possible disruptions that may affect complex supply chain networks (Gurnani, Ray, and Mehrotra 2012). In this paper, we only consider disruptions that cause a supplier to not fulfil the order altogether. Cases of partial fulfilment (Dada, Petruzzi, and Schwarz 2007; Li, Sethi, and Zhang 2013a, 2013b) or late fulfilment (Dolgui and Ould-Louly 2002) are not discussed.

The literature on supply chain disruptions is very much diversified. Snyder et al. (2012) provide an excellent review of the literature on supply chain disruption management. They discuss nearly 150 scholarly papers on topics including evaluation of supply disruptions, strategic decisions, sourcing decisions, contracts and incentives, inventory and facility location. The supply chain disruption literature on which our work is based can be classified into four streams: (1) price-dependent demand, (2) competition among buyers, (3) default by an unreliable supplier and (4) contingent sourcing strategy to overcome supply disruptions.

The first line of the literature focuses on inventory decisions with price-dependent demands. Early on, most of the operations management literature dealt with pricing in inventory/capacity management and focused on a single product with perfectly reliable supply. Whitin (1995) and Mills (1959, 1962) were among the first who considered endogenous prices in inventory/capacity models. Dada, Petruzzi, and Schwarz (2007) and Li, Sethi, and Zhang (2013a) study sourcing and pricing decisions of a firm ordering from several suppliers and facing a price-dependent demand. They show that for a firm, *cost is the order qualifier while reliability is the order winner* in choosing a supplier. Ha and Tong (2008) study two competing firms under contracts with suppliers and facing a demand that is linear in price. The market demand can be low or high. Shou, Huang, and Li (2009) also use a price-dependent linear demand to study management of supply chains under disruption.

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The second stream studies competing suppliers and buyers exposed to supply disruptions. Shou, Huang, and Li (2009) discuss two competing supply chains subject to supply uncertainties. The retailers engage in a Cournot competition by determining the quantities to be ordered from their exclusive suppliers. They examine the decisions of the suppliers and the retailers at three different levels: operational, design and strategic. They find that supply chain coordination may or may not result in positive gains for the supply chain, depending on the extent of the supply risk. Li, Wang, and Cheng (2010) examine the sourcing strategy of a retailer and the pricing strategies of two suppliers in a supply chain facing supply disruptions. They use the spot market as a perfectly reliable contingent supplier and characterise the sourcing strategy of the retailer in both centralised and decentralised systems. Tang and Kouvelis (2011) study the benefits from supplier diversification for two dual-sourcing competing buyers. The authors conclude that buyers should never choose to use a common supplier, because the increased correlation between the delivered quantities leads to a decrease in the buyers' profits.

Wang, Gilland, and Tomlin (2010) compare the effectiveness of dual-sourcing and reliability improvement strategies. They show that a combined strategy of contingent dual-sourcing and reliability improvement can provide significant value if suppliers are very unreliable and/or capacity is low relative to demand. Tomlin (2009) study how supply learning affects sourcing and inventory policies when firms adopt contingent dual-sourcing or single-sourcing strategies. He analyses a Bayesian model (via distribution updating) of 'supply learning' to investigate how supply learning affects both sourcing and inventory decisions in single-sourcing and dual-sourcing models.

We consider a supply chain with two Suppliers, U and R, and two competing Buyers, C and S, selling the same products in the market. The competition between the buyers is modelled as a Cournot quantity game. Supplier U is unreliable and Supplier R is reliable, but more expensive. This setting is also used by Tomlin (2006) and Chopra, Reinhardt, and Mohan (2007); see also Kazaz (2004) and Tomlin and Snyder (2006). We assume that Firm C places an order with Supplier U first, and will place an emergency order with Supplier R if Supplier U cannot deliver the order. For expositional brevity, we refer to this as a contingent dual-sourcing strategy (CDSS). Firm S purchases only from Supplier R. We refer to this as a sole sourcing strategy, or SSS for short. We characterise the optimal quantity and the expected profit for each manufacturer under different cases, and obtain important managerial insights.

There have been a number of real-life instances of CDSS reported in the literature. For example, in response to the air traffic disruption resulting from 9/11, Chrysler temporarily shipped components by ground from the US to their Dodge Ram assembly plant in Mexico (Tomlin 2006). The primary benefit of CDSS over maintaining safety stock is that the cost is incurred only in the event of a supply disruption. Although CDSS has been used by many firms, there is a lack of research on the impact of such a strategy on supply chains. Is it always cost-effective? Under what conditions is it a dominant strategy to manage supply disruptions? How does CDSS affect firms' decisions under competition? We add to the literature on CDSS by investigating the strategy in a duopoly setting. Although, the timing of supply disruptions is unpredictable, firms have control over procurement times. Buyer C can place his order before or after, or simultaneously with another buyer.

The remainder of this paper is organised as follows. In Section 2, the duopoly model is introduced and formulated. In Section 3, we analyse various possible games in the duopoly setting under different cases. For each case, we obtain the optimal order quantities and expected profits for both buyers. We also obtain some properties of the equilibrium orders as well as the associated expected profits. In Section 4, we compare the equilibria in the games studied in Section 3. Numerical computations to get further insights are presented in Section 5. In Section 6, we study two extensions of the model studied in Section 3. In Sections 6.1 and 6.2 we discuss the equilibrium long-run sourcing strategies of two asymmetric and symmetric firms that can choose between SSS and CDSS. We study the impact of capacity reservation by the CDSS firm to secure supply from Supplier R in Section 6.3. Finally, Section 7 summarises the main contributions of our work and suggests future research directions.

2. The model

To investigate the impact of supply disruptions on competing buyers, we consider a supply chain as depicted in Figure 1. There are two Buyers, C and S, who procure parts from Suppliers U and R. Buyers C and S process these parts into substitutable products to be sold at the same price in the market to end consumers. The product has a short life cycle and is sold in a single-selling season. Supplier U is unreliable and Supplier R is reliable, and the unit costs charged by them are c_1 and c_2, respectively, where $c_2 > c_1 > 0$ for regular orders. In normal situations (*without supply disruptions*), C buys products from Supplier U. If Supplier U defaults, C places an emergency order with Supplier R at a unit cost of c_3 where $c_3 \geq c_2$ on account of S being Supplier R's preferred customer. We should note here that an issue of whether the scenario we model in this section could arise endogenously could be raised. To answer this, we provide in Section 6.1 an example of two asymmetric firms C and S with $c_3 > c_2$ in which Firm C chooses CDSS and Firm S chooses SSS when the reliability of Supplier U is at a medium level. Of course, here we study the case in the general parameter settings of $c_3 \geq c_2$ and $\alpha \in [0, 1]$.

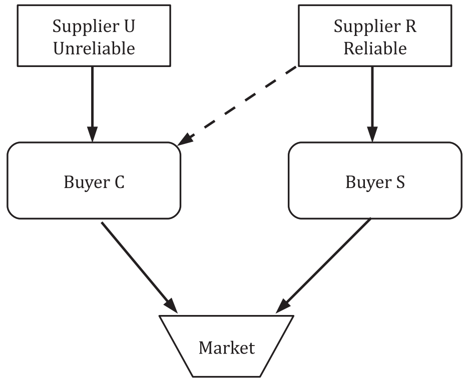


Figure 1. Sourcing model.

We let the binary random variable X, taking values in $\{0, 1\}$, denote the supply state of Supplier U. When $X = 1$, Supplier U can deliver all that is ordered, and when $X = 0$, he can deliver nothing. We assume that α denotes the probability that $X = 1$. Supplier R, on the other hand, can always deliver whatever is ordered.

We model the manufacturers to engage in a quantity competition. C orders Q_1 from Supplier U and S orders Q_2 from Supplier R. If Supplier U defaults, then C orders Q_{e0} from Supplier R, otherwise none is ordered, i.e. $Q_{e1} = 0$. The price p of the products is determined by a linear demand function $p(S) = a - S$, where S is the total product quantity delivered to Buyers C and S, and $a > 0$ denotes the potential market size. We assume that a is sufficiently large to ensure that $p > 0$; as shown in the next section, we must assume that $a > 3c_3 - 2c_2 + c_1$. The linear demand function substantially simplifies the analysis, and has been extensively used in the literature; see e.g. Ha and Tong (2008), and Li, Sethi, and Zhang (2013b), and references therein.

In order to compare the equilibrium quantities ordered and the expected profits for C and S, we study a benchmark supply chain when Supplier R is the only supplier and both C and S order from it.

2.1 *The benchmark profits for C and S under SSS*

When both buyers order from Supplier R, there are three possible ordering sequences: (i) both order simultaneously, (ii) C orders first and (iii) S orders first.

In Case (i), the buyers play a Nash game. The optimal order quantities for C and S are $(a+c_2-2c_3)/3$ and $(a+c_3-2c_2)/3$, respectively. Accordingly, the maximum profits for C and S are $(a + c_2 - 2c_3)^2/9$ and $(a + c_3 - 2c_2)^2/9$, respectively.

In Case (ii), when C orders before S does, it becomes a Stackelberg game with C as the leader and S as the follower. The optimal order quantity and profit for C are $(a + c_2 - 2c_3)/2$ and $(a + c_2 - 2c_3)^2/8$, respectively. The optimal order quantity and the profit for S are $(a - 3c_2 + 2c_3)/4$ and $(a - 3c_2 + 2c_3)^2/16$, respectively.

Case (iii) with S ordering first is symmetric to Case (ii), and so the optimal order quantity and the equilibrium profit, respectively, for C are $(a - 3c_3 + 2c_2)/4$ and $(a - 3c_3 + 2c_2)^2/16$ and for S are $(a + c_3 - 2c_2)/2$ and $(a + c_3 - 2c_2)^2/8$.

The maximum possible profits for C and S over all three cases when ordering from Supplier R are $(a + c_2 - 2c_3)^2/8$ and $(a + c_3 - 2c_2)^2/8$, respectively. These will serve as the benchmark profits for assessing the benefits for C and S, respectively, in all games considered in this paper.

3. Analysis of the games with dual sourcing

There are a number of events taking place in our model – C orders, S orders, the supply state realizes and C places an emergency order if Supplier U is disrupted. Depending on the occurrence of these events, there arise seven different cases as shown in Figures 2–8. We let Q_1^j, Q_2^j, Q_e^j, Π_C^j, Π_S^j, S^j denote C's order quantity, S's order quantity, C's emergency order quantity, C's expected profit, S's expected profit and the total market supply in Case j in equilibrium, $j = 1, 2, \ldots, 7$. We also define the notation Q_{e0} as the emergency order when $x = 0$ and $Q_{e1} = 0$ when $x = 1$. For the multi-stage games under consideration, the concepts we use are feedback Nash and feedback Stackelberg equilibria as defined, e.g. in Basar and Olsder (1999) and Bensoussan, Chen, and Sethi (2014). These can be obtained by a backward induction procedure, and are time consistent or subgame perfect. We analyse the cases one by one and find the quantities ordered and the expected

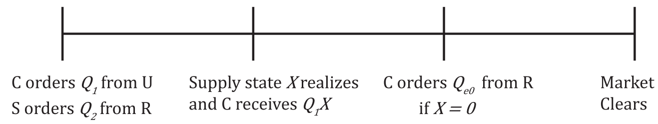


Figure 2. Sequence of events for Case 1.

profits in equilibrium in each case. We also discuss the sensitivity of the results with respect to the model parameters in each case. We obtain managerial insights indicating how the results change with respect to the model parameters in each case.

3.1 *Case 1*

The sequence of events in this case is shown in Figure 2.

In this case, C and S simultaneously order Q_1 and Q_2 from Suppliers U and R, respectively, in the first stage. Then the supply state X realises and C receives the quantity Q_1X from Supplier U and S receives the quantity Q_2 from Supplier R. In the next stage, C places an emergency order Q_{e0} if $X = 0$. After that, the market clears and the profits of C and S are realised.

The game played is a multi-stage game in which C places an emergency order from Supplier R, after both C and S have ordered Q_1 and Q_2 simultaneously from Suppliers U and R, respectively, after the supply state X is realised. We use backward induction to obtain the equilibrium solution. That is, C's emergency order quantity response will be given as a feedback function $q_e(Q_1, Q_2, x)$, where x is the realisation of X. If Supplier U does not default, i.e. $x = 1$, then clearly $q_e(Q_1, Q_2, 1) = 0$. However, when $x = 0$, C will maximise his profit to obtain $q_e(Q_1, Q_2, 0)$, i.e. $\max_{q_e}[(a - Q_2 - q_e - c_3)q_e]$. By solving this, we obtain the best response of buyer C, given Q_1 and Q_2 as $q_e(Q_1, Q_2, 0) = \dfrac{(a - Q_2 - c_3)}{2}$. Thus, the entire feedback policy is

$$q_e(Q_1, Q_2, x) = \begin{cases} Q_{e0} = \dfrac{a - Q_2 - c_3}{2} & \text{if } x = 0, \\ Q_{e1} = 0 & \text{if } x = 1. \end{cases} \tag{1}$$

Next, we solve the Nash game between C and S, knowing C's emergency order quantity reaction function. That is, C and S obtain Q_1 and Q_2 simultaneously by maximising their respective expected profits. In view of (1), therefore, we have the following simultaneous maximisation problems:

$$\max_{Q_1}\left[\alpha\,(a - Q_1 - Q_2 - c_1)\,Q_1 + (1-\alpha)\left(\frac{a - Q_2 - c_3}{2}\right)^2\right], \tag{2}$$

$$\max_{Q_2}\left[\alpha\,(a - Q_1 - Q_2 - c_2)\,Q_2 + (1-\alpha)\left(a - \frac{a - Q_2 - c_3}{2} - Q_2 - c_2\right)Q_2\right]. \tag{3}$$

Solving the first-order condition gives,

$$Q_1^{1*} = \frac{(1+\alpha)(a - 2c_1) + 2c_2 - (1-\alpha)c_3}{2\,(\alpha+2)} \text{ and } Q_2^{1*} = \frac{a + \alpha c_1 - 2c_2 + (1-\alpha)c_3}{\alpha + 2}. \tag{4}$$

These are indeed the optimal order quantities since the objective functions (2) and (3) are jointly strictly concave in Q_1 and Q_2. The equilibrium in Case 1 can now be expressed as the triple $(Q_1^{1*}, Q_2^{1*}, Q_e^{1*})$, where Q_e^{1*} is the random variable $Q_e^{1*} = q_e(Q_1^{1*}, Q_2^{1*}, X)$. Inserting $Q_e^{1*} = 0$ when $X = 1$ and $Q_{e0} = \dfrac{(1+\alpha)a - \alpha c_1 + 2c_2 - 3c_3}{2\,(\alpha+2)}$ into the objective functions (2) and (3), we obtain the equilibrium-expected profits for C and S, respectively, as

$$E\left(\Pi_{\mathrm{C}}^{1}\right) = \frac{\alpha\,[(1+\alpha)(a - 2c_1) + 2c_2 - (1-\alpha)c_3]^2 + (1-\alpha)\,[(1+\alpha)a - \alpha c_1 + 2c_2 - 3c_3]^2}{4\,(\alpha+2)^2},$$

$$E\left(\Pi_{\mathrm{S}}^{1}\right) = \frac{(1+\alpha)\,(a + \alpha c_1 - 2c_2 + (1-\alpha)c_3)^2}{2(\alpha+2)^2}.$$

The expected total market output is

$$E\left(S^1\right) = \alpha\left(Q_1^{1*} + Q_2^{1*}\right) + (1-\alpha)\left(Q_e^{1*} + Q_2^{1*}\right) = \frac{(3+\alpha)a - \alpha(1+\alpha)c_1 - 2c_2 - (1-\alpha^2)c_3}{2(\alpha+2)}.$$

PROPOSITION 1 *Q_1^{1*} increases in α and c_2 and decreases in c_1 and c_3; Q_2^{1*} decreases in α and c_2 and increases in c_1 and c_3; and Q_e^{1*} increases in α and decreases in c_1, c_2 and c_3, almost surely.*

Proposition 1 says that when c_2 increases, Supplier U has a higher cost advantage over supplier R. So, C increases the quantity ordered from supplier U. When c_1 increases, the cost advantage for C reduces and he buys less from supplier U. An increase in supplier U's reliability α means that C has a higher chance of realising the cost advantage over S, and therefore, C buys more and S buys less from Supplier R.

PROPOSITION 2 *In the equilibrium in Case 1, the expected total market output decreases in c_1, c_2 and c_3, and increases in α if $2c_2 + \left(\alpha^2 + 4\alpha + 1\right)c_3 > a + \left(\alpha^2 + 4\alpha + 2\right)c_1$ and does not increase otherwise.*

The expected market price in Case 1 is $E\left(p^1\right) = a - E\left(S^1\right)$, and it is straightforward to obtain the results about the expected market price from the expected total market supply.

3.2 *Case 2*

The sequence of events in this case is shown in Figure 3.

Here, C orders Q_1 first from Supplier U and then S orders Q_2 from Supplier R. Then the supply state X realises and C receives Q_1X from Supplier U and S receives Q_2 from Supplier R. Following this, C places an emergency order Q_e with Supplier R depending on the realisation of X. After that, the market clears and the profits of C and S are realised.

In the first stage, C leads and S follows in placing the orders Q_1 and Q_2, respectively. In the second stage, S leads with his order Q_2 and C follows by his emergency order Q_e. Therefore, C's order quantity response is the same feedback function $q_e(Q_1, Q_2, x)$ as given in (1). Anticipating this and knowing C's order Q_1, S maximises his expected profit. Thus, S's problem is:

$$\max_{Q_2}\left[\alpha\left(a - Q_1 - Q_2 - c_2\right)Q_2 + (1-\alpha)\left(\frac{a - Q_2 + c_3 - 2c_2}{2}\right)Q_2\right]. \quad (5)$$

Using the first-order condition, we obtain S's best response function

$$q_2(Q_1) = \frac{(1+\alpha)a - 2c_2 + (1-\alpha)c_3 - 2\alpha Q_1}{2(1+\alpha)}, \quad (6)$$

which C uses in the first stage to obtain his order Q_1. For this, C maximises his expected profit:

$$\max_{Q_1}\left[\alpha\left(a - Q_1 - q_2(Q_1) - c_1\right)Q_1 + (1-\alpha)\left(\frac{a - q_2(Q_1) - c_3}{2}\right)^2\right], \quad (7)$$

and obtains

$$Q_1^{2*} = \frac{a(\alpha+1)(\alpha+3) - 4(\alpha+1)^2c_1 + 2\alpha(c_2+c_3) + 6c_2 + 3\alpha^2c_3 - 5c_3}{2(4+3\alpha+\alpha^2)}. \quad (8)$$

Plugging (8) in (6), we get

$$Q_2^{2*} = \frac{2\left(a + (\alpha+1)\alpha c_1 - (\alpha+2)c_2 + \alpha^2(-c_3) + c_3\right)}{4+3\alpha+\alpha^2}. \quad (9)$$

These are indeed the optimal order quantities since the objective functions (5) and (7) are jointly strictly concave in Q_1 and Q_2. Thus, the equilibrium triple in Case 2 is given by $(Q_1^{2*}, Q_2^{2*}, Q_e^{2*})$, where $Q_e^{2*} = q_e\left(Q_1^{2*}, Q_2^{2*}, 0\right)$ with q_e as defined in (1). In particular, when $X = 0$, the emergency order quantity $q_e\left(Q_1^{2*}, Q_2^{2*}, 0\right) = Q_{e0}^2 = \frac{a(\alpha+1)(\alpha+2) + \alpha(-2(\alpha+1)c_1 + 2c_2 + (\alpha-3)c_3) + 4c_2 - 6c_3}{2(4+3\alpha+\alpha^2)}$. Inserting (8) and (9) into the objective functions (5) and (7), we obtain the equilibrium -expected profits for C and S, respectively, as

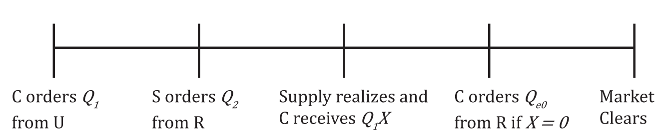


Figure 3. Sequence of events for Case 2.

$$E\left(\Pi_C^2\right) = \frac{1}{4(4+3\alpha+\alpha^2)}\big[\alpha^2\big(a^2 + a(6c_3 - 8c_1) + 8c_1^2 - 4c_1(c_2+c_3) + c_3(4c_2 - 3c_3)\big)$$
$$+2\alpha\big(a^2 - a(3c_1 - 2c_2 + c_3) + 2c_1^2 + c_1(5c_3 - 6c_2) + 4c_3(c_2 - c_3)\big)$$
$$+2\alpha^3(c_3 - c_1)(a - 2c_1 + c_3) + (a + 2c_2 - 3c_3)^2\big],$$

$$E\left(\Pi_S^2\right) = \frac{2(\alpha+1)\left(a + (\alpha+1)\alpha c_1 - (\alpha+2)c_2 + \alpha^2(-c_3) + c_3\right)^2}{(4+3\alpha+\alpha^2)^2}.$$

The expected total market output $E\left(S^2\right)$ is given by

$$E\left(S^2\right) = Q_2^{2*} + \alpha Q_1^{2*} + (1-\alpha)\, Q_{e0}^{2*} = \frac{a(\alpha(\alpha+2)+3) + (\alpha+1)^2((\alpha-1)c_3 - \alpha c_1) - 2c_2}{4+3\alpha+\alpha^2}.$$

PROPOSITION 3 *The expected total market supply $E\left(S^2\right)$ decreases in c_1, c_2 and c_3, and increases in α if*

$$6c_2 - c_3 + a(-1+\alpha(2+\alpha)) - c_1(1+\alpha)(4+\alpha(12+\alpha(5+\alpha))) + \alpha(4c_2 + c_3(10+\alpha(16+\alpha(6+\alpha)))) > 0.$$

3.3 *Case 3*

The sequence of events in this case is shown in Figure 4.

In this case, C first orders Q_1 and then the supply state X realises and C receives Q_1X from Supplier U. Following this, C places an emergency order Q_{e0} with Supplier R if $X = 0$. After that, S orders Q_2 from Supplier R. Finally, the market clears and the profits of C and S are realised.

The game is a multi-stage game in which S follows by deciding Q_2 given C's two-stage decision of ordering Q_1 and then Q_{e0} if $X = 0$. Therefore, S's profit maximisation problem is:

$$\max_{Q_2} \left[(a - Q_1x - Q_e(Q_1, x) - Q_2 - c_2)\, Q_2\right], \tag{10}$$

where $Q_e(Q_1, x)$ is the emergency order quantity by C when $X = x$. We can solve (10) to obtain the reaction function of S:

$$q_2(Q_1, Q_e(Q_1, x)) = \frac{a - Q_1x - Q_e(Q_1, x) - c_2}{2}. \tag{11}$$

The emergency order $Q_{e1} = 0$ when $x = 1$, so we only need to solve for Q_1 and Q_{e0}. This can be done by solving C's expected profit maximisation problem:

$$\max_{Q_1, Q_{e0}} \left[\alpha\left(a - Q_1 - \frac{a - Q_1 - c_2}{2} - c_1\right)Q_1 + (1-\alpha)\left(a - Q_{e0} - \frac{a - Q_{e0} - c_2}{2} - c_3\right)Q_{e0}\right]. \tag{12}$$

This gives

$$Q_{e0}^{3*} = \frac{a + c_2 - 2c_3}{2} \text{ and } Q_1^{3*} = \frac{a + c_2 - 2c_1}{2}. \tag{13}$$

Moreover, we can express C's emergency order feedback policy as

$$Q_e^{3*}(Q_1, x) = \begin{cases} \dfrac{a + c_2 - 2c_3}{2} & \text{if } x = 0, \\ 0 & \text{if } x = 1. \end{cases} \tag{14}$$

Substituting (13) and (14) in (11), we get S's optimal equilibrium order quantity Q_2^{3*} as

$$Q_2^{3*}(x) = \begin{cases} \dfrac{a - 3c_2 + 2c_3}{4} & \text{if } x = 0, \\ \dfrac{a - 3c_2 + 2c_1}{4} & \text{if } x = 1. \end{cases} \tag{15}$$

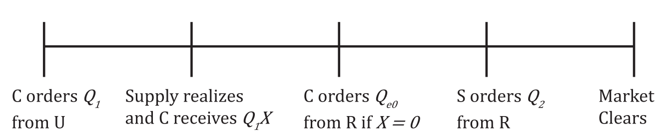


Figure 4. Sequence of events for Case 3.

Inserting (13)–(15) into the objective functions (10) and (12), we obtain the equilibrium-expected profits for C and S as

$$E\left(\Pi_C^3\right) = \alpha\frac{(a+c_2-2c_1)^2}{8} + (1-\alpha)\frac{(a+c_2-2c_3)^2}{8},$$
$$E\left(\Pi_S^3\right) = \alpha\frac{(a-3c_2+2c_1)^2}{16} + (1-\alpha)\frac{(a-3c_2+2c_3)^2}{16}.$$

It is easy to see that $E\left(\Pi_C^3\right) > E\left(\Pi_S^3\right)$ when $c_2 = c_3$. Also, $E\left(\Pi_C^3\right) < E\left(\Pi_S^3\right)$ for large values of c_3 and small α. The expected total market output is

$$E\left(S^3\right) = \alpha\left(\frac{a+c_2-2c_1}{2} + \frac{a-3c_2+2c_1}{4}\right) + (1-\alpha)\left(\frac{a+c_2-2c_3}{2} + \frac{a-3c_2+2c_3}{4}\right)$$
$$= \frac{3a-2\alpha c_1-c_2-2(1-\alpha)c_3}{4}.$$

PROPOSITION 4 *At the equilibrium of Case 3, the expected total market output $E\left(S^3\right)$ increases in α and decreases in c_1, c_2 and c_3. The expected market price $E\left(p^4\right)$ decreases in α and increases in c_1, c_2 and c_3.*

3.4 *Case 4*

The sequence of events in this case is shown in Figure 5.

Thus in this case, C orders Q_1 first, then supply state X realises, and C receives Q_1 if Supplier U does not default. Next, C and S simultaneously order Q_e and Q_2 from Supplier R, respectively. After that, the market clears and the profits of C and S are realised.

In the second stage, C and S order Q_e and Q_2 simultaneously from Supplier R, after C has ordered Q_1 from Supplier U in the first stage and the supply state X is realised at the end of the first stage. Note that if Supplier U defaults, i.e. $X = 0$, the game at the second stage is a simple quantity competition game, where C and S order $(a+c_2-2c_3)/3$ and $(a+c_3-2c_2)/3$ from Supplier R, respectively. When $X = 1$, C does not place an emergency order and S's profit maximisation problem is:

$$\max_{Q_2}\,(a-Q_1-Q_2-c_2)\,Q_2. \tag{16}$$

The first-order condition gives

$$q_2(Q_1, 1) = \frac{a-Q_1-c_2}{2}. \tag{17}$$

Therefore, we obtain C's and S's order quantity responses in the equilibrium as

$$q_e^{4*}(Q_1, x) = \begin{cases} \dfrac{a+c_2-2c_3}{3} & \text{if } x = 0, \\ 0 & \text{if } x = 1, \end{cases} \tag{18}$$

$$Q_2^{4*}(Q_1, x) = \begin{cases} \dfrac{a-2c_2+c_3}{3} & \text{if } x = 0, \\ \dfrac{a-Q_1-c_2}{2} & \text{if } x = 1. \end{cases} \tag{19}$$

Next, we solve the game at the first stage in which C anticipates his own and S's order quantity responses in the second stage given by (18) and (19), respectively, and orders Q_1 that maximises his expected profit, i.e.

$$\max_{Q_1}\left[\alpha\left(\frac{a+c_2-2c_1-Q_1}{2}\right)Q_1 + (1-\alpha)\left(\frac{a+c_2-2c_3}{3}\right)^2\right]. \tag{20}$$

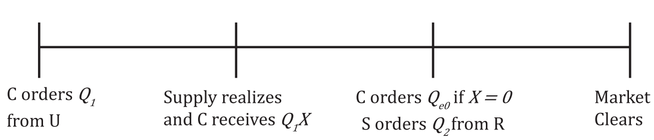


Figure 5. Sequence of events for Case 4.

From the first-order condition, we solve for Q_1^{4*}, C's optimal order quantity in stage 1, as

$$Q_1^{4*} = \frac{a + c_2 - 2c_1}{2}. \tag{21}$$

Substituting (21) into (19), we have $Q_2^{4*} = \dfrac{a - 3c_2 + 2c_1}{4}$, if $x = 1$. Using the equilibrium order quantities, we can now derive the expected profits for C and S as

$$E\left(\Pi_C^4\right) = \alpha \frac{(a + c_2 - 2c_1)^2}{8} + (1 - \alpha)\, \frac{(a + c_2 - 2c_3)^2}{9},$$
$$E\left(\Pi_S^4\right) = \alpha \frac{(a - 3c_2 + 2c_1)^2}{16} + (1 - \alpha)\, \frac{(a - 2c_2 + c_3)^2}{9}.$$

It is easy to see that $E\left(\Pi_C^4\right) > E\left(\Pi_S^4\right)$ when $c_2 = c_3$. Also, $E\left(\Pi_C^4\right) < E\left(\Pi_S^4\right)$ for large values of c_3 and small α. The expected total market output is

$$E\left(S^4\right) = \alpha\left(\frac{a + c_2 - 2c_1}{2} + \frac{a - 3c_2 + 2c_1}{4}\right) + (1 - \alpha)\left(\frac{a + c_2 - 2c_3}{3} + \frac{a - 2c_2 + c_3}{3}\right),$$
$$= \alpha\left(\frac{3a - c_2 - 2c_1}{4}\right) + (1 - \alpha)\left(\frac{2a - c_2 - c_3}{3}\right).$$

PROPOSITION 5 *In the equilibrium of Case 4, the expected total market output $E\left(S^4\right)$ increases in α and decreases in c_1, c_2 and c_3. Consequently, the expected market price $E\left(p^4\right)$ decreases in α but increases in c_1, c_2 and c_3.*

3.5 *Case 5*

The sequence of events in this case is shown in Figure 6.

Here, C orders Q_1 first and then the supply state X realises, and then C receives $Q_1 X$ from Supplier U. After that S orders Q_2 from Supplier R. Following this, C places an emergency order Q_{e0} with Supplier R if $X = 0$. Finally, the market clears and the profits of C and S are realised.

The game has two stages. In the first stage, C leads and S follows in placing the orders Q_1 and Q_2, respectively. In the second stage, C follows and S leads when they place orders Q_e and Q_2, respectively. Therefore, in the second stage, C's order quantity response will be the feedback function $q_e(Q_1, Q_2, x)$ given in (1).

Next, we solve the game between C and S in the second stage where S anticipates C's emergency order $Q_e(Q_1, Q_2, X)$ and maximises his expected profit. From (1), we obtain S's expected profit maximisation problem as

$$\max_{Q_2} E\left[(a - Q_1 x - Q_e(Q_1, Q_2, X) - c_2)\, Q_2\right]. \tag{22}$$

Using (1), we obtain the order quantity response for S using the first-order condition

$$q_2(Q_1, X) = \frac{a - c_2}{2} - \frac{Q_1 X}{X + 1}. \tag{23}$$

From this and (1), C's emergency order quantity Q_{e0}^{4*} when $X = 0$ is

$$Q_{e0}^{5*} = \frac{a + c_2 - 2c_3}{4}. \tag{24}$$

Now, we solve the game in the first stage where C factors in S's order quantity $Q_2 = q_2(Q_1, X)$ given by (23), and orders Q_1 from Supplier U that maximises his expected profit, i.e.

$$\max_{Q_1}\left[\alpha\left(\frac{a - Q_1 - 2c_1 + c_2}{2}\right) Q_1 + (1 - \alpha)\left(\frac{a + c_2 - 2c_3}{4}\right)^2\right]. \tag{25}$$

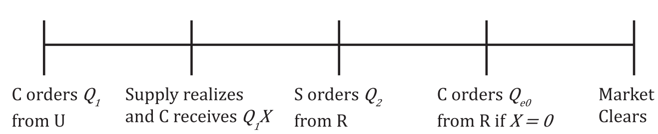


Figure 6. Sequence of events for Case 5.

Solving this, gives

$$Q_1^{5*} = \frac{a + c_2 - 2c_1}{2}. \tag{26}$$

Plugging (26) in (23), we obtain

$$Q_2^{5*}(x) = \begin{cases} \dfrac{a - c_2}{2} & \text{if } x = 0, \\ \dfrac{a - 3c_2 + 2c_1}{4} & \text{if } x = 1. \end{cases} \tag{27}$$

The expected profits for C and S are

$$E\left(\Pi_C^5\right) = \alpha \frac{(a + c_2 - 2c_1)^2}{8} + (1 - \alpha)\frac{(a + c_2 - 2c_3)^2}{16},$$
$$E\left(\Pi_S^5\right) = \alpha \frac{(a - 3c_2 + 2c_1)^2}{16} + (1 - \alpha)\frac{(a - c_2)^2}{8}.$$

The expected total market output is

$$\begin{aligned} E\left(S^5\right) &= \alpha\left(\frac{a + c_2 - 2c_1}{2} + \frac{a - 3c_2 + 2c_1}{4}\right) + (1 - \alpha)\left(\frac{a + c_2 - 2c_3}{4} + \frac{a - c_2}{2}\right) \\ &= \alpha\left(\frac{3a - c_2 - 2c_1}{4}\right) + (1 - \alpha)\left(\frac{3a - c_2 - 2c_3}{4}\right) \\ &= \frac{3a - 2\alpha c_1 - c_2 - 2(1 - \alpha)c_3}{4}. \end{aligned}$$

PROPOSITION 6 *In the equilibrium, $E\left(S^5\right)$ increases in α and decreases in c_1, c_2, and c_3. The expected market price $E\left(p^5\right)$ decreases in α and increases in c_1, c_2, and c_3.*

3.6 *Case 6*

The sequence of events in this case is shown in Figure 7.

This results in a multi-stage game where S leads by ordering Q_2 and C follows with orders Q_1 in the first stage. In the second stage, C's order quantity response function will be the same feedback function $q_e(Q_1, Q_2, x)$ given by (1), since the game is identical to the game in Case 1. And we are left with the problem of solving only the first stage of the game where C orders Q_1 by maximising his expected profit:

$$\max_{Q_1}\left[\alpha\,(a - Q_1 - Q_2 - c_1)\,Q_1 + (1 - \alpha)\,\frac{(a - Q_2 - c_3)^2}{4}\right]. \tag{28}$$

From the first-order condition, we have

$$q_1(Q_2) = \frac{a - Q_2 - c_1}{2}. \tag{29}$$

In the first stage, S leads and orders Q_2 from Supplier R by anticipating C's order quantity responses Q_1 and Q_e. Therefore, S's expected profit maximisation problem in view of (1) and (29) is given as

$$\max_{Q_2}\left[\alpha\frac{(a - Q_2 + c_1 - 2c_2)}{2}Q_2 + (1 - \alpha)\,\frac{(a - Q_2 - 2c_2 + c_3)}{2}Q_2\right]. \tag{30}$$

Using the first-order condition, we obtain the equilibrium order quantity

$$Q_2^{6*} = \frac{a + \alpha c_1 - 2c_2 + (1 - \alpha)c_3}{2}. \tag{31}$$

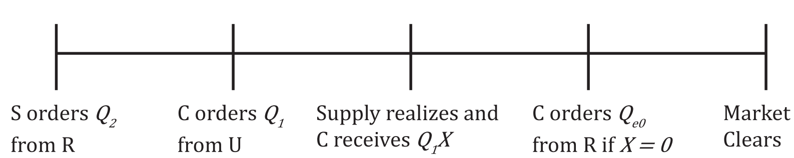


Figure 7. Sequence of events in Case 6.

From (1), (29), and (31), we obtain

$$Q_1^{6*} = \frac{a - (2+\alpha)c_1 + 2c_2 - (1-\alpha)c_3}{4}, \tag{32}$$

$$q_e^{6*}(x) = \begin{cases} \dfrac{a - \alpha c_1 + 2c_2 - (3-\alpha)c_3}{4} & \text{if } x = 0, \\ 0 & \text{if } x = 1. \end{cases} \tag{33}$$

Using these order quantities, we find the expected profits for C and S as

$$\begin{aligned} E\left(\Pi_{\mathrm{C}}^{6}\right) &= \alpha \left(\frac{a - (\alpha+2)c_1 + 2c_2 - (1-\alpha)c_3}{4}\right)^2 + (1-\alpha)\left(\frac{a - \alpha c_1 + 2c_2 - (3-\alpha)c_3}{4}\right)^2 \\ &= \frac{1}{16}\left((a + 2c_2 - 3c_3)^2 + 2(3a - 2c_1 + 6c_2 - 7c_3)(-c_1 + c_3)\alpha + 5(c_1 - c_3)^2\alpha^2\right), \\ E\left(\Pi_{\mathrm{S}}^{6}\right) &= \left\{\alpha\left(\frac{a - 3c_2 + \alpha c_2 - \alpha c_1 + 2c_1}{4}\right) + (1-\alpha)\left(\frac{a - c_2 + \alpha c_2 - \alpha c_1}{4}\right)\right\}\left(\frac{a - c_2 - \alpha c_2 + \alpha c_1}{2}\right) \\ &= \frac{1}{8}(a + \alpha c_1 - 2c_2 + (1-\alpha)c_3)^2. \end{aligned}$$

Here, a direct comparison of C's and S's expected profit is not straightforward. So, we resort to numerical analysis in Section 5 to compare the profits. The expected total market output is

$$\begin{aligned} E\left(\mathrm{S}^6\right) &= \alpha\left(\frac{a - (2+\alpha)c_1 + 2c_2 - (1-\alpha)c_3}{4} + \frac{a + \alpha c_1 - 2c_2 + (1-\alpha)c_3}{2}\right) \\ &\quad + (1-\alpha)\left(\frac{a - \alpha c_1 + 2c_2 - (3-\alpha)c_3}{4} + \frac{a + \alpha c_1 - 2c_2 + (1-\alpha)c_3}{2}\right) \\ &= \frac{1}{4}\left(3a - \alpha c_1 - 2c_2 - (1-\alpha)c_3\right). \end{aligned}$$

PROPOSITION 7 *In the equilibrium of Case 6, the* $E\left(\mathrm{S}^6\right)$ *increases in* α *and decreases in* c_1, c_2, *and* c_3. p^6 *decreases in* α *and increases in* c_1, c_2, *and* c_3.

3.7 *Case 7*

The sequence of events in Case 7 is shown in Figure 8.

In the last case, S orders Q_2 first from Supplier R and then the supply state X is realised. Following this, C orders Q_1 from Supplier U if $X = 1$ or places an emergency order of Q_e with Supplier R if $X = 0$. After that, the market clears and the profits of C and S are realised.

In the multi-stage game played here, S leads by ordering Q_2 from Supplier R, and C follows with orders Q_1 from Supplier U if $x = 1$, or Q_{e0} from Supplier R, when $x = 0$. C's order quantity response will be the same feedback function $q_e(Q_1, Q_2, x)$ as given by (1). Next, we solve the game when the realised supply state is $x = 1$. C, being the follower, orders Q_1 from Supplier U to maximise his expected profit, i.e.

$$\max_{Q_1}[(a - Q_1 - Q_2 - c_1)\, Q_1]. \tag{34}$$

From the first-order condition, we obtain

$$q_1(Q_2) = \frac{a - Q_2 - c_1}{2}. \tag{35}$$

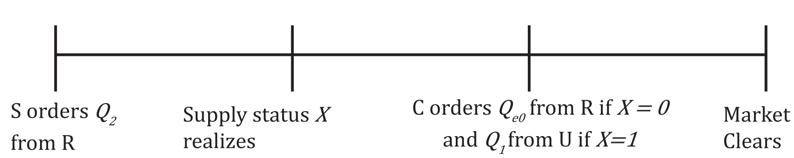


Figure 8. Sequence of events for Case 7.

S anticipates C's order quantities (1) and (35), and orders Q_2 from Supplier R that maximises his expected profit:

$$\max_{Q_2}\left[\alpha\frac{(a-Q_2+c_1-2c_2)}{2}Q_2+(1-\alpha)\,\frac{(a-Q_2-2c_2+c_3)}{2}Q_2\right]. \tag{36}$$

The first-order condition gives S's equilibrium order quantity Q_2^{7*} for the first-stage game as

$$Q_2^{7*}=\frac{a-2c_2+\alpha c_1+(1-\alpha)c_3}{2}. \tag{37}$$

From (1), (35), and (37), we obtain the equilibrium order quantities $Q_1^{7*}=\dfrac{a-(2+\alpha)c_1+2c_2-(1-\alpha)c_3}{4}$ and $Q_e^{7*}=\dfrac{a-\alpha c_1+2c_2-(3-\alpha)c_3}{4}$. The expected profits of C and S are

$$\begin{aligned}
E\left(\Pi_{\mathrm{C}}^{7}\right)&=\alpha\left(\frac{a-(\alpha+2)c_1+2c_2-(1-\alpha)c_3}{4}\right)^2+(1-\alpha)\left(\frac{a-\alpha c_1+2c_2-(3-\alpha)c_3}{4}\right)^2\\
&=\frac{1}{16}\left((a+2c_2-3c_3)^2+2(3a-2c_1+6c_2-7c_3)(-c_1+c_3)\alpha+5(c_1-c_3)^2\alpha^2\right),\\
E\left(\Pi_{\mathrm{S}}^{7}\right)&=\left\{\alpha\left(\frac{a-3c_2+\alpha c_2-\alpha c_1+2c_1}{4}\right)+(1-\alpha)\left(\frac{a-c_2+\alpha c_2-\alpha c_1}{4}\right)\right\}\left(\frac{a-c_2-\alpha c_2+\alpha c_1}{2}\right)\\
&=\frac{1}{8}(a+\alpha c_1-2c_2+(1-\alpha)c_3)^2.
\end{aligned}$$

The expected total market output is identical to that in Case 6. Since $E\left(\mathrm{S}^7\right)=E\left(\mathrm{S}^6\right)$, all other equilibrium results in Case 7 are identical to those in Case 6.

4. Equilibrium profit comparisons

In the previous section, we derived explicit expressions for the Equilibrium-expected profits of C and S in all cases. In Section 2.1, we obtained the benchmark profit. In this section, we compare the expected profits of C and S in all seven cases and also to the benchmark profit.

PROPOSITION 8 (i) *Expected profits for C satisfy:* $E\left(\Pi_{\mathrm{C}}^2\right)>E\left(\Pi_{\mathrm{C}}^1\right)>E\left(\Pi_{\mathrm{C}}^6\right)=E\left(\Pi_{\mathrm{C}}^7\right)$, $E\left(\Pi_{\mathrm{C}}^3\right)>E\left(\Pi_{\mathrm{C}}^4\right)>E\left(\Pi_{\mathrm{C}}^5\right)$, *and* $E\left(\Pi_{\mathrm{C}}^5\right)>E\left(\Pi_{\mathrm{C}}^1\right)$.

(ii) *Expected profits for S satisfy:* $E\left(\Pi_{\mathrm{S}}^2\right)<E\left(\Pi_{\mathrm{S}}^1\right)<E\left(\Pi_{\mathrm{S}}^6\right)=E\left(\Pi_{\mathrm{S}}^7\right)$, $E\left(\Pi_{\mathrm{S}}^3\right)<E\left(\Pi_{\mathrm{S}}^4\right)<E\left(\Pi_{\mathrm{S}}^5\right)$, *and* $E\left(\Pi_{\mathrm{S}}^1\right)>E\left(\Pi_{\mathrm{S}}^4\right)$.

(iii) *The expected profits of C satisfy* $E(\Pi_{\mathrm{C}}^i)\le\Pi_{\mathrm{C}}^0$ *for* $i\neq 3$ *and* $E(\Pi_{\mathrm{C}}^3)>\Pi_{\mathrm{C}}^0=(a+c_2-2c_3)^2/8$, *the benchmark profit. The expected profits of S in all cases satisfy* $E(\Pi_{\mathrm{S}}^i)\le\Pi_{\mathrm{C}}^0$. *Finally,* $E\left(\Pi_{\mathrm{C}}^3\right)>E\left(\Pi_{\mathrm{S}}^3\right)$.

As we go from Case 3 to Case 5, we see from Propositions 8 (i) and (ii) that the time when S places orders. This confers an advantage to S, and consequently his expected profit increases and the expected profit of C decreases.

In Proposition 8(iii), we compare the expected profits of C with the benchmark profit Π_{C}^0. We observe that only in Case 3, C has a higher profit than its benchmark profit. The intuitive explanation is that purchasing from an unreliable supplier is better for someone with first-mover advantage and when the supply state is realised before the competitor orders. Similarly, the equilibrium-expected profits of S are less than the benchmark profit in all cases. Therefore, the profit of C is always more than the profit of S in Case 3. This is further corroborated by the numerical computations in Section 5.

PROPOSITION 9 *If* $\alpha\ge 0.5$ *and* $c_2=c_3$, *then* $E\left(\Pi_{\mathrm{C}}^5\right)>E\left(\Pi_{\mathrm{S}}^5\right)$.

4.1 *Comparative statics*

The comparative statics of the equilibrium-expected profits of C and S are summarised in Table 1. We see that the equilibrium-expected profits for both C and S are monotone in α, c_1 and c_3. C's profit is not monotone with c_2, and S's profit is decreasing with c_2. In the absence of monotonicity of $E(\Pi_{\mathrm{C}}^i)$ with c_2, we conjecture that when the supply from 1 is highly reliable, $E(\Pi_{\mathrm{C}}^i)$ increases in c_2, and when the supply from 1 is highly unreliable, $E(\Pi_{\mathrm{C}}^i)$ decreases in c_2. The intuitive explanation for this conjecture relies on the trade-off between the supply costs c_1, c_2 and c_3 to C and S and the reliability α of Supplier U. With a higher reliability of Supplier U, the chance of order fulfilment is higher, i.e. C purchases at the cheaper price c_1 from Supplier U with a higher chance. Subsequently, C earns more in the market due to his cost advantage over S since $c_2\ge c_1$.

Table 1. Comparative statics of Equilibrium-expected profits under cases $i = 1, \ldots, 7$. Here, '↑' indicates increasing, '↓' indicates decreasing and '↕' indicates non-monotonicity.

$\frac{\partial E\left(\Pi_C^i\right)}{\partial \alpha}$	$\frac{\partial E\left(\Pi_C^i\right)}{\partial c_1, c_3}$	$\frac{\partial E\left(\Pi_C^i\right)}{\partial c_2}$	$\frac{\partial E\left(\Pi_S^i\right)}{\partial \alpha}$	$\frac{\partial E\left(\Pi_S^i\right)}{\partial c_1, c_3}$	$\frac{\partial E\left(\Pi_S^i\right)}{\partial c_2}$
↑	↓	↕	↓	↑	↓

Table 2. Comparative statics of equilibrium order quantities. Here, ↑ indicates increasing, ↓ indicates decreasing, and ↕ indicates unchanging. ↑ (↕) and ↓ (↕) indicates increasing and decreasing in c_1, respectively, and unchanging in c_3. ↕ (↓) indicates unchanging in c_1 and decreasing in c_3.

Order quantity →	Q_1^{i*}			Q_2^{i*}			Q_{e0}^{i*}		
Parameter →	α	$c_1(c_3)$	c_2	α	$c_1(c_3)$	c_2	α	$c_1(c_3)$	c_2
Case 1	↑	↓	↑	↓	↑	↓	↑	↓	↑
Case 2	↑	↓	↑	↓	↑	↓	↑	↓	↑
Case 3	↕	↓ (↕)	↑	↓	↑	↓	↕	↕ (↓)	↑
Case 4	↕	↓ (↕)	↑	↕	↑	↓	↕	↕ (↓)	↑
Case 5	↕	↓ (↕)	↑	↑	↑ (↕)	↓	↕	↕ (↓)	↑
Case 6	↓	↓	↑	↑	↑	↓	↓	↓	↑
Case 7	↓	↓	↑	↑	↑	↓	↓	↓	↑

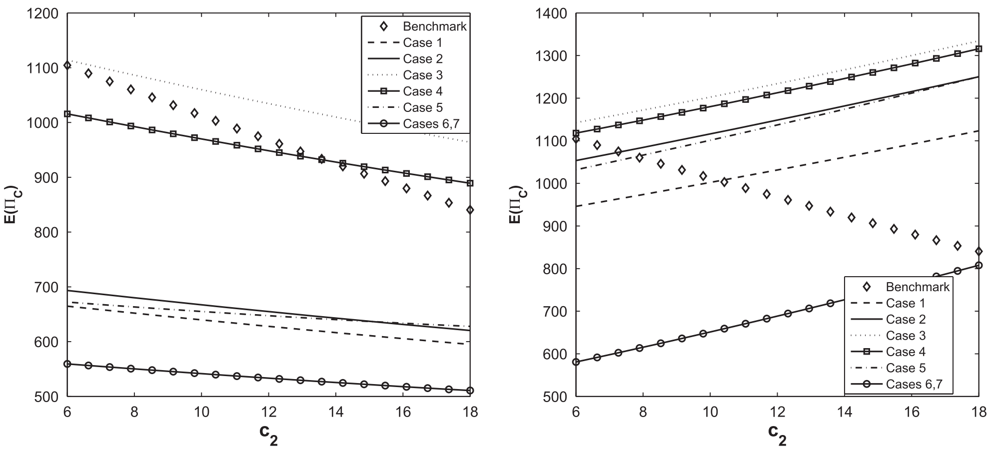
We now summarise the comparative statics of the equilibrium order quantities for C and S in all seven cases. The most interesting takeaway from Table 2 is the impact of reliability on the order quantities. We see that Q_1^{i*} and Q_{e0}^{i*} are increasing in α in Cases 1 and 2, where C orders from Supplier U before or at the same time as S does from Supplier R. In Cases 6 and 7, S orders from Supplier R before C does from Suppliers U and R. Therefore, both Q_1^{i*} and Q_{e0}^{i*} are decreasing in α in Cases 6 and 7. Similarly, we see that Q_2^{i*} is decreasing in α in Cases 1 and 2 and increasing in α for Cases 6 and 7, respectively. We also see from Table 2 that the quantity ordered by a firm is always non-increasing in its own procurement costs and non-decreasing in its competitor's procurement costs.

5. Computational analysis

In this section, we present the results obtained from computations that confirm as well as add to the conclusions drawn in Section 3. We compute the expected equilibrium profits for both C and S in all cases by varying c_1, c_2, c_3 and α. Since the equilibrium profits of C as well as S are identical in Cases 6 and 7, we analyse them together as 'cases 6, 7'. The profits of C and S are monotone in the same direction with both c_1 and c_3, so we assume $c_3 = c_2$ when analysing the impacts of c_1 and c_2 on the equilibrium-expected profits of C and S. In particular, we first illustrate the impact of combinations of α, c_1 and c_2 on the equilibrium-expected profits of C and S and on the equilibrium total order quantities in all cases. We let $a = 100$ in all computations.

5.1 *Impact of supplier costs and reliability on profit of C*

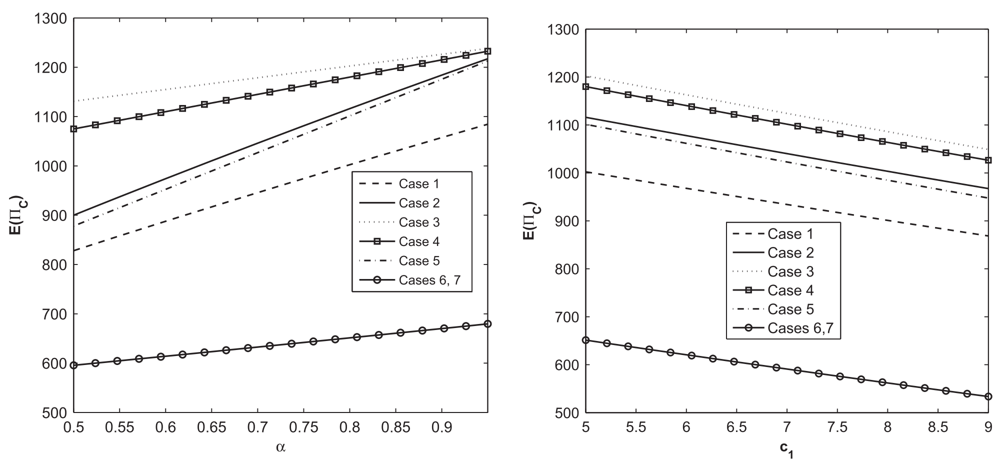
Figure 9(a) and (b) shows the equilibrium-expected profit of C with respect to c_2 for low and high values of α, respectively. For computing $E(\Pi_C)$, we let $c_1 = 5$ and vary $c_2 \in [6, 18]$. First, we see in Figure 9(a) that when the supplier is highly unreliable, say when $\alpha = 0.2$, $E(\Pi_C)$ is decreasing in c_2. On the other hand, we see in Figure 9(b) that when the supplier is highly reliable (say $\alpha = 0.8$) then an increase in the cost for S leads to an increase in the profit of C. This observation is in line with the conjecture proposed in Section 4. This dependence of the monotonicity of $E(\Pi_C)$ with c_2 on α can be explained as follows: when Supplier U is highly reliable, there is less of a chance that C will order from Supplier R at the higher cost $c_3 \geq c_2$. This increases C's profitability. We also see from Figure 9 that the profits for C satisfy $E\left(\Pi_C^3\right) > E\left(\Pi_C^4\right) > E\left(\Pi_C^{2,5}\right) > E\left(\Pi_C^1\right) > E\left(\Pi_C^6\right) = E\left(\Pi_C^7\right)$. This is in line with Proposition 9(i). This shows that adopting CDSS is beneficial for C as it increases his profits.



(a) $\alpha = 0.2$.

(b) $\alpha = 0.8$.

Figure 9. Variation of $E(\Pi_C)$ with respect to c_2 and α.



(a) Variation with α.

(b) Variation with c_1.

Figure 10. Variation of $E(\Pi_C)$ with respect to α and the wholesale cost c_1.

Figure 10(a) shows the equilibrium-expected profit of C in all cases with α. We let $c_1 = 5, c_2 = 10$, and vary $\alpha \in [0.5, 0.95]$. We observe that the profit of C always increases in α, which is also shown in Table 1. The cause of this increase in profit of C is due to the increase in reliability of Supplier U, which gives C a cost advantage over S as Supplier U is cheaper than 2. Therefore, the expected equilibrium profit of C increases in α. Figure 10(b) shows how c_1 affects the profit of C. In these numerical computations, we fix $\alpha = 0.8,\ c_2 = 10$, and vary $c_1 \in [5, 9]$. We see that the profit of C decreases with an increase in c_1, which is also seen in Table 1. This decrease in the profit of C is due to the decrease in the benefit of ordering from the cheaper supplier. Accordingly, C orders less from Supplier U and the profit of C decreases. The orders between the expected profits of C under different cases remain intact in Figure 10(a) and (b), as the order of profits in Figure 9(a) and (b).

5.2 *Impact of supplier costs and reliability on profit of S*

To study the variation of the equilibrium-expected profit of S with c_2, we fix $c_1 = 5, \alpha = 0.8$ and vary $c_2 \in [6, 20]$. Figure 11 shows that $E(\Pi_S)$ decreases in c_2, in all cases. This confirms with the comparative statics reported in Table 1. $E(\Pi_S)$ in all cases satisfy: $E\left(\Pi_S^3\right) < E\left(\Pi_S^2\right) < E\left(\Pi_S^4\right) < E\left(\Pi_S^5\right) < E\left(\Pi_S^1\right) < E\left(\Pi_S^6\right) = E\left(\Pi_S^7\right)$. We find that $E(\Pi_S)$ increases from Case 3 through Case 6, as the procurement decision of S from Supplier R is later and later in time. We also see that the profits of S in all cases are lower than the benchmark profit, even when he decides before C, e.g. S orders from Supplier R before C does from Supplier U in Cases 6 and 7.

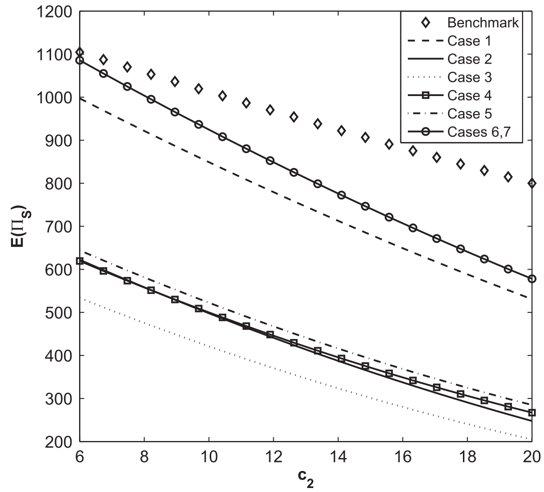
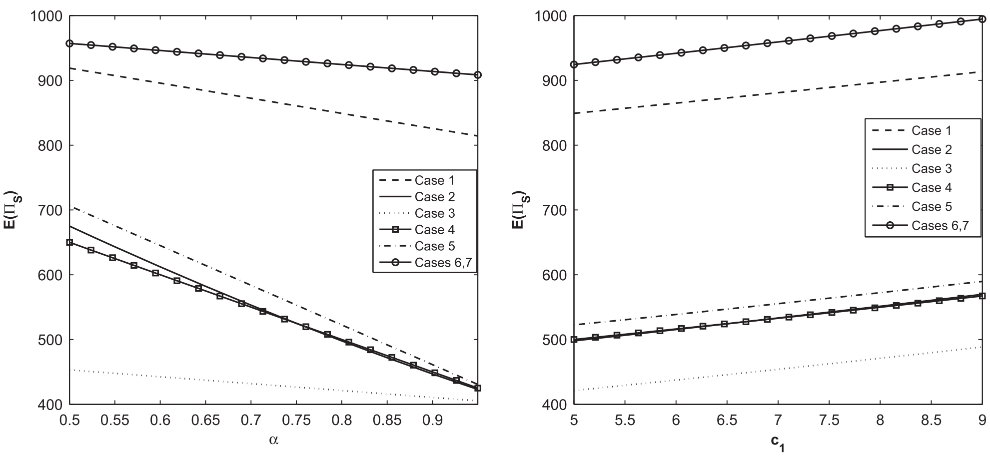


Figure 11. Variation of equilibrium profit $E(\Pi_S)$ with respect to the cost c_2.



(a) Variation with α. (b) Variation with c_1.

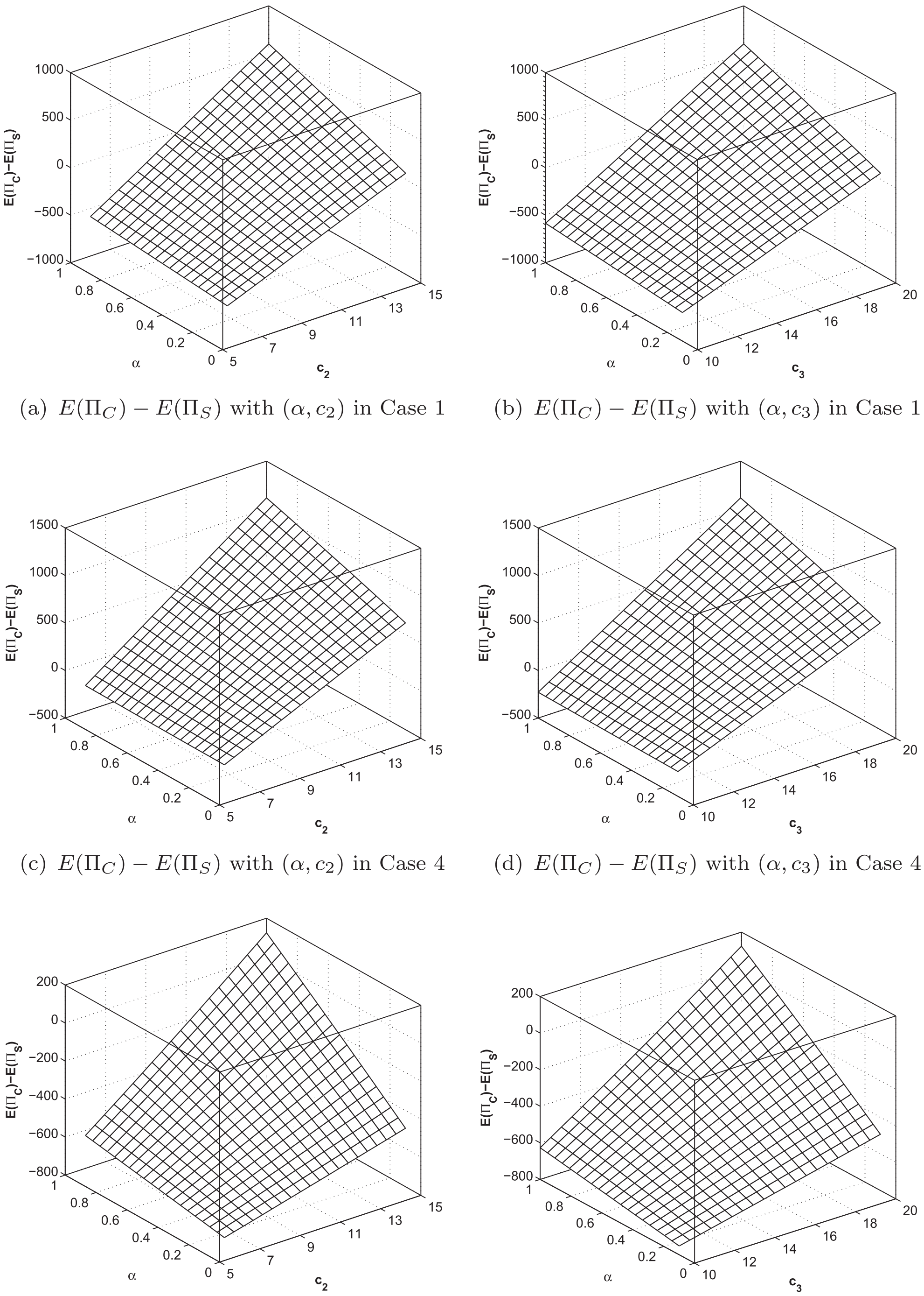
Figure 12. Variation of equilibrium profit of S with respect to reliability α and the wholesale cost c_1.

Figure 12(a) shows the impact of the reliability of Supplier U on the profits of S. We fix $c_1 = 5$, $c_2 = 10$ and vary $\alpha \in [0.5, 0.95]$. We observe from Figure 12(a) that $E(\Pi_S)$ decreases in α in all cases. Further, to study the impact of Supplier U's cost c_1 on $E(\Pi_S)$, we fix $\alpha = 0.8$, $c_2 = 10$ and vary $c_1 \in [5, 9]$. From Figure 12(b), we see that $E(\Pi_S)$ increases in c_1 in all cases. We also see that the equilibrium-expected profit of S satisfies: $E\left(\Pi_S^3\right) < E\left(\Pi_S^4\right) \le E\left(\Pi_S^2\right) < E\left(\Pi_S^5\right) < E\left(\Pi_S^1\right) < E\left(\Pi_S^6\right) = E\left(\Pi_S^7\right)$.

5.3 *Impact of contingent sourcing on profits*

Now we study the impact of CDSS strategy on the equilibrium-expected profits of C and S. To be able to study the impact of CDSS on the profits of C and S, we compute the difference between the profits of C and S, who adopt CDSS and SSS strategies, respectively. The difference between the profits of C and S is $E\left(\Pi_C^i\right) - E\left(\Pi_S^i\right)$. Therefore, CDSS does better than SSS only if $E(\Pi_C) - E(\Pi_S) > 0$. Note that, in Section 4 we established $E\left(\Pi_C^3\right) > E\left(\Pi_S^3\right)$. Therefore, CDSS is better than SSS when C and S operate in a situation identical to Case 3. We also compare cases $i \in \{1, 4, 6\}$ using computational analysis. For these computations, we use $c_1 = 5$, $\alpha \in [0.1, 0.95]$, $c_2 \in [6, 15]$ and $c_3 \in [10, 20]$.

In Figure 13(a)–(f), we see that there is no clear winner between CDSS and SSS. If α is low and c_2 is not too large compared to c_1, then SSS is better than CDSS for S and worse for C. If α is high and c_2 is larger than c_1, then CDSS is better than SSS for C and worse for S.



(a) $E(\Pi_C) - E(\Pi_S)$ with (α, c_2) in Case 1 (b) $E(\Pi_C) - E(\Pi_S)$ with (α, c_3) in Case 1

(c) $E(\Pi_C) - E(\Pi_S)$ with (α, c_2) in Case 4 (d) $E(\Pi_C) - E(\Pi_S)$ with (α, c_3) in Case 4

(e) $E(\Pi_C) - E(\Pi_S)$ with (α, c_2) in Case 6 (f) $E(\Pi_C) - E(\Pi_S)$ with (α, c_3) in Case 6

Figure 13. $E(\Pi_C) - E(\Pi_S)$ with respect to (α, c_2) , (α, c_3) under cases 1,4 and 6.

5.4 *Impact of reliability level on total market output*

To study the impact of reliability on the total market output $E\left(S^i\right)$, we set $c_1 = 5, c_2 = 10$ and vary $\alpha \in [0.1, 1.0]$. Figure 14 shows the variation of $E\left(S^i\right)$ with respect to α. We see that $E\left(S^i\right)$ has no clear monotonicity in all cases. $E\left(S^i\right)$ is not necessarily increasing with the reliability of Supplier U. Surprisingly, in Case 1, it is strictly decreasing. The expected total market output in Case 1 satisfies Proposition 2, i.e. when $2c_2 + \left(\alpha^2 + 4\alpha + 1\right) c_3 > a + \left(\alpha^2 + 4\alpha + 2\right) c_1$, then $E\left(S^1\right)$ is

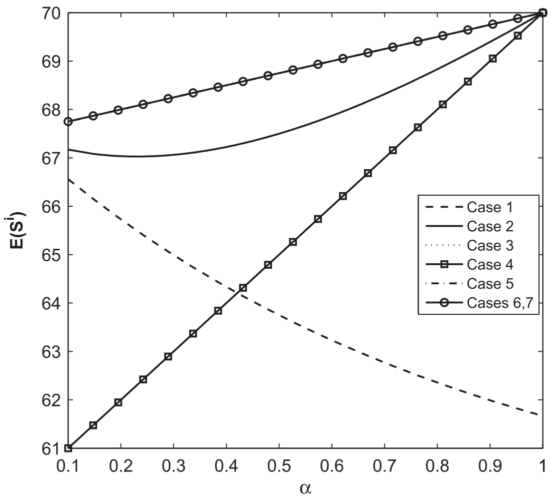


Figure 14. $E\left(S^i\right)$ with α.

decreasing with α. Therefore, Figure 14 validates the Proposition using the specific parameter values we chose for these computations.

6. Extensions

In this section, we discuss three important extensions. In the first two, we extend the model presented in Section 3 to study endogenous sourcing strategies for asymmetric and symmetric firms in the market. The asymmetric firms, C and S, differ in the sense that $c_3 > c_2$. In Section 6.1, we provide examples that show that, depending on the parameters, the firms should choose different strategies or the same strategies in equilibrium. Moreover, these examples indicate that depending on the cost difference, if the reliability of Supplier U is high, medium or low, both firms choose CDSS, Firm C chooses CDSS and Firm S chooses SSS, or both firms choose SSS, respectively. The last choice results interestingly from a prisoner's dilemma. In Section 6.2, the Firms C and S are symmetric with $c_3 = c_2$. Now both firms choose CDSS if Supplier U is highly reliable. Otherwise, and once again, both are in a prisoner's dilemma and choose SSS. In Section 6.3, we study the impact of capacity reservation by C with the reliable supplier R on the equilibrium order quantities.

6.1 *Endogenous sourcing strategy: asymmetric firms*

Thus far, we have studied the firms that choose order quantities in the short run with specified sourcing strategies. However, in the long run, firms can decide their sourcing strategies by maximising their respective profits. So here, we study two asymmetric firms C and S, interested in choosing their optimal sourcing strategies from CDSS or SSS and deciding simultaneously their order quantities, as studied in Section 3.1. There are three cases to consider: (a) C chooses CDSS and S chooses SSS; (b) both choose SSS; and (c) both choose CDSS. The profits of C and S in Case (a) are given in Section 3.1. In Case (b), C and S order from Supplier R, and the optimal order quantity is $\dfrac{a-c_2}{3}$. The expected profit for each firm is $\dfrac{(a-c_2)^2}{9}$. Case (c) is a two-stage game between the firms. In the first stage, C and S order from Supplier U, and in the second stage, they order from Supplier R if there is disruption. Accordingly, the optimal order quantity from Supplier U is $\dfrac{a-c_1}{3}$ and from Supplier R is $\dfrac{a-c_3}{3}$. The expected profit for each firm is $\dfrac{\alpha(a-c_1)^2+(1-\alpha)(a-c_3)^2}{9}$. Since $c_3 > c_1$, this profit increases with α. Therefore, there is a threshold $\alpha_r = \dfrac{(2a-c_2-c_3)(c_3-c_2)}{(2a-c_1-c_2)(c_2-c_1)+(2a-c_2-c_3)(c_3-c_2)}$, such that the profit in Case (c) is higher than the profit in Case (b) with $\alpha > \alpha_r$. Furthermore, when $c_2 = c_3$, the profit in Case (c) is always higher than in Case (b), and vice-versa if $c_1 = c_2$.

A reason for asymmetry may be due to the location of these firms in relation to Supplier R. Say, for example, that Firm S is located nearer to Supplier R than Firm C is. Then it is reasonable to assume that $c_2 < c_3$. This difference in the costs

Buyer S

Buyer C	Expected Profit	CDSS	SSS
	CDSS	**944.4, 982.2**	926.2, 889.0
	SSS	849.0, 1002.3	711.1, 900.0

Both firms choose CDSS when reliability is high with $\alpha = 0.8$.

Buyer S

Buyer C	Expected Profit	CDSS	SSS
	CDSS	886.1, 961.7	**742.5, 980.0**
	SSS	895.7, 887.7	711.7, 900.0

Asymmetric sourcing equilibrium when $\alpha = 0.6$.

Buyer S

Buyer C	Expected Profit	CDSS	SSS
	CDSS	798.6, 930.8	473.6, 1120.6
	SSS	962.4, 704.1	**711.1, 900.0**

Both firms choose SSS when reliability is low with $\alpha = 0.3$. This game is the classical 'prisoner's dilemma'.

Figure 15. Equilibrium sourcing strategies for buyers with different reliabilities of supplier U.

could also arise if S has a closer relationship with Supplier R. Furthermore, for convenience of not having too many cases to deal with, we assume that the firms are symmetric in all other aspects.

In Figure 15, we present the pay-offs for C and S in a 2×2 matrix when they choose CDSS or SSS with different reliability levels of the supplier U. We set $a = 100$, $c_1 = 5$, $c_2 = 10$, $c_3 = 20$ and vary $\alpha \in \{0.3, 0.6, 0.8\}$. We call these levels low, medium and high, respectively, for the purpose of this discussion. We observe that in the equilibrium, both choose CDSS when the reliability is high, and choose SSS, by playing a prisoner's dilemma game, when the reliability is low. However, at the medium reliability level, C chooses CDSS and S chooses SSS. It is this last setting that motivates the formulation of our model in Section 3 where we assume Firms C and S to follow CDSS and SSS, respectively.

6.2 *Endogenous sourcing strategy: symmetric firms*

We now study symmetric firms and show that they, as expected, choose the same sourcing strategy in equilibrium as well as identify the situations where they would choose CDSS or SSS. At a low-reliability level of Supplier U, we find that C and S play a prisoner's dilemma game, whereas they play a 'stag-hunt' game (Harsanyi and Selten 1988) at the medium reliability level.

In Figure 16, we present the pay-offs for C and S when they choose CDSS or SSS with different reliability levels of supplier U. We set $a = 100$, $c_1 = 5$, $c_2 = c_3 = 10$ and vary $\alpha \in \{0.3, 0.5, 0.8\}$. As in Section 6.1, the profit of the firms with CDSS is higher than the profit with SSS. So in equilibrium, both firms choose CDSS when α is high and choose SSS, by way of the prisoner's dilemma, when α is low. However, when the reliability level is medium, we see that there are two equilibria in pure strategies like in a stag-hunt game. If both firms collude to choose CDSS before the game, the pay-offs are (951.4, 951.4) when $\alpha = 0.5$, which leads to a Pareto dominant equilibrium. Otherwise they both fall prey to the prisoner's dilemma.

6.3 *Capacity reservation by Firm C*

In some cases, a contingent dual sourcing firm may reserve capacity with a reliable supplier to mitigate disruption. Here we study the impact of capacity reservation by Firm C with Supplier R on the equilibrium order quantities and the profits of the firms. Let C pay c_r to Supplier R for each unit of reserved capacity. This fee is charged as an insurance against a possible disruption. R charges c_3^r for each unit purchased after disruption. Clearly, if $c_3^r = c_3$, then no capacity will be reserved and

Buyer S

Buyer C	Expected Profit	CDSS	SSS
	CDSS	**982.2, 982.2**	1002.3, 849.0
	0.3 SSS	849.0, 1002.3	900.0, 900.0

Both firms choose CDSS when reliability is high with $\alpha = 0.8$.

Buyer S

Buyer C	Expected Profit	CDSS	SSS
	CDSS	**951.4, 951.4**	828.1, 918.8
	SSS	918.8, 828.1	**900.0, 900.0**

Two equilibria in pure strategies where both buyers choose either CDSS or SSS, $\alpha = 0.5$.

Buyer S

Buyer C	Expected Profit	CDSS	SSS
	CDSS	930.8, 930.8	704.1, 962.4
	SSS	962.4, 704.1	**900.0, 900.0**

Both firms choose SSS when reliability is low with $\alpha = 0.3$. This game is the classical 'prisoner's dilemma'.

Figure 16. Equilibrium sourcing strategies for buyers with different reliabilities of supplier U. The pay-offs for Buyers C and S are represented by Π_C, Π_S.

the entire analysis in Section 3 holds. To avoid trivial case in this section, we will assume that $c_r > 0$ and $c_3^r < c_3$. One would expect that the firm may reserve capacity if the reservation cost is not too high and/or the difference $c_3 - c_3^r$ is not too small. For our analysis therefore, we will fix c_3^r and obtain a threshold T_r such that the firm will not reserve capacity if $c_r > T_r$.

Proposition 10 *Whenever a firm chooses to reserve capacity, the amount reserved will be equal to the emergency order quantity. Moreover, the capacity reserved decreases in c_3^r and remains unchanged in c_r.*

We next analyze the cases presented in Section 3, now with Firm C having an option to reserve capacity with Supplier R.

6.3.1 *Case 1 with capacity reservation*

The sequence of events in Case 1 is shown in Figure 17.

In words, C and S simultaneously order Q_1 and Q_2 from Suppliers U and R, respectively, and C reserves a capacity K with Supplier R at the same time it orders from Supplier U. Then the supply state X realizes, C receives the quantity Q_1X from Supplier U, and S receives the quantity Q_2 from Supplier R. In the next stage, C places an emergency order Q_{e0} if $X = 0$. Then the market clears and the profits of C and S are realized.

What is played is a multi-stage game in which C places an emergency order from Supplier R upon the realization of the supply state X, whereas, before this realization, both C and S have already ordered Q_1 and Q_2 simultaneously from Suppliers U and R, respectively, and C has reserved capacity K from Supplier R. We use backward induction to obtain the equilibrium solution. That is, C's emergency order quantity response will be given as a feedback function $q_e(Q_1, Q_2, x)$, where x is the realization of X. If Supplier U does not default, i.e. $x = 1$, then clearly $q_e(Q_1, Q_2, 1) = 0$. However, when $x = 0$, C will

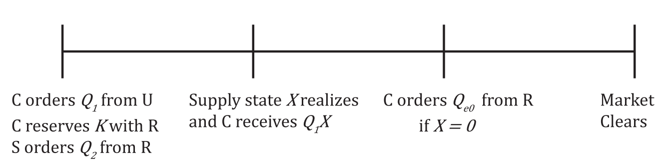


Figure 17. Sequence of events for Case 1 with capacity reservation.

maximize his profit to obtain $q_e(Q_1, Q_2, 0)$, i.e. $\max_{q_e}[(a - Q_2 - q_e - c_3^r)\, q_e]$. By solving this, we obtain the best response of buyer C given Q_1 and Q_2 as $q_e(Q_1, Q_2, 0) = \dfrac{(a - Q_2 - c_3^r)}{2}$. Thus, the entire feedback policy is

$$q_e(Q_1, Q_2, x) = \begin{cases} Q_{e0} = \dfrac{a - Q_2 - c_3^r}{2} & \text{if } x = 0, \\ Q_{e1} = 0 & \text{if } x = 1. \end{cases} \tag{38}$$

Next we solve the Nash game between C and S, knowing C's emergency order quantity reaction function. That is, C and S obtain Q_1, K and Q_2 simultaneously by maximizing their respective expected profits. In view of (38), therefore, we have the following simultaneous maximization problems:

$$\max_{Q_1, K} \left[\alpha\,(a - Q_1 - Q_2 - c_1)\, Q_1 - c_r K + (1-\alpha) \left(\frac{a - Q_2 - c_3^r}{2} \right)^2 \right], \tag{39}$$

$$\max_{Q_2} \left[\alpha\,(a - Q_1 - Q_2 - c_2)\, Q_2 + (1-\alpha) \left(a - \frac{a - Q_2 - c_3^r}{2} - Q_2 - c_2 \right) Q_2 \right]. \tag{40}$$

Solving the first-order condition with $K \geq Q_{e0}$ gives

$$K^{1*} = Q_{e0}\,,\; Q_1^{1*} = \frac{(1+\alpha)(a - 2c_1) + 2c_2 - (1-\alpha)c_3^r}{2\,(\alpha+2)} \text{ and } Q_2^{1*} = \frac{a + \alpha c_1 - 2c_2 + (1-\alpha)c_3^r}{\alpha+2}. \tag{41}$$

These are indeed the optimal order quantities since the objective functions (39) and (40) are jointly strictly concave in K, Q_1 and Q_2. The equilibrium can now be expressed as the triple $(Q_1^{1*}, Q_2^{1*}, Q_e^{1*})$, where Q_e^{1*} is the random variable $Q_e^{1*} = q_e(Q_1^{1*}, Q_2^{1*}, X)$. Inserting $Q_e^{1*} = 0$ when $X = 1$ and $K^{1*} = Q_{e0} = \dfrac{(1+\alpha)a - \alpha c_1 + 2c_2 - 3c_3^r}{2\,(\alpha+2)}$ into the objective functions (39) and (40), we obtain the equilibrium-expected profits for C and S, respectively, as

$$E\left(\Pi_C^1\right) = \frac{1}{4\,(\alpha+2)^2} \Big[\alpha \left[(1+\alpha)(a-2c_1) + 2c_2 - (1-\alpha)c_3^r \right]^2 + (1-\alpha) \left[(1+\alpha)a - \alpha c_1 + 2c_2 - 3c_3^r \right]^2$$
$$-2(\alpha+2)c_r \left[(1+\alpha)a - \alpha c_1 - 2c_2 - 3c_3^r \right] \Big],$$

$$E\left(\Pi_S^1\right) = \frac{(1+\alpha)\left(a + \alpha c_1 - 2c_2 + (1-\alpha)c_3^r\right)^2}{2(\alpha+2)^2}.$$

The expected total market output is

$$E\left(S^1\right) = \alpha\left(Q_1^{1*} + Q_2^{1*}\right) + (1-\alpha)\left(Q_e^{1*} + Q_2^{1*}\right) = \frac{(3+\alpha)a - \alpha(1+\alpha)c_1 - 2c_2 - (1-\alpha^2)c_3^r}{2\,(\alpha+2)}.$$

By comparing the profit of firm C in Section 3.1 with the profit above, we find that the firm will reserve capacity with Supplier R when

$$c_r < T_r = \frac{1}{2(\alpha+2)(\alpha a + a - \alpha c_1 - 2c_2 - 3c_3^r)} \Big\{ -\alpha(\alpha(a - 2c_1 + c_3) + a - 2(c_1 + c_2) - c_3)^2$$
$$+ (\alpha-1)(\alpha a + a - \alpha c_1 + 2c_2 - 3c_3)^2 - (\alpha-1)(\alpha a + a - \alpha c_1 + 2c_2 - 3c_3^r)^2$$
$$+ \alpha(a(\alpha+1) - 2(\alpha+1)c_1 + 2c_2 + (\alpha-1)c_3^r)^2 \Big\},$$

and Firm C will not reserve capacity with Supplier R when c_r exceeds T_r.

6.3.2 *Other cases with capacity reservation*

Analyses of the other cases are similar to the analysis for Case 1 above. We can derive T_r in each case, and if c_r is below the corresponding threshold, then the quantities ordered by firms C and S can be obtained simply by replacing c_3 with c_3^r in the formulas obtained in each case. The profit of firm C in each case is obtained by replacing c_3 with c_3^r in the derived formula and then subtracting the cost $c_r Q_{e0}$ of reserving the capacity $K = Q_{e0}$ at a cost c_r from the profit. We now summarize the threshold T_r for Cases 2–7.

Case 2:

$$T_r = \frac{1}{2(a(\alpha+1)(\alpha+2)+\alpha(-2(\alpha+1)c_1+2c_2+(\alpha-3)c_3^r)+4c_2-6c_3^r)}$$
$$\Big\{-2\alpha^3\left(a^2-a(3c_1-2c_2+c_3)+2c_1^2+c_1(5c_3-6c_2)+4c_3(c_2-c_3)\right)$$
$$\left(a^2+a(6c_3-8c_1)+8c_1^2-4c_1(c_2+c_3)+c_3(4c_2-3c_3)\right)$$
$$+2\alpha^3\left(a^2-a(3c_1-2c_2+c_3^r)+2c_1^2+c_1(5c_3^r-6c_2)+4c_3^r(c_2-c_3^r)\right)$$
$$\left(a^2+a(6c_3^r-8c_1)+8c_1^2-4c_1(c_2+c_3^r)+c_3^r(4c_2-3c_3^r)\right)$$
$$+2\alpha^3(c_1-c_3)(a-2c_1+c_3)+2\alpha^3(c_3^r-c_1)(a-2c_1+c_3^r)-(a+2c_2-3c_3)^2+(a+2c_2-3c_3^r)^2\Big\}.$$

Cases 3 and 5: $T_r = \dfrac{(1-\alpha)(c_3-c_3^r)(a+c_2-c_3-c_3^r)}{a+c_2-2c_3^r}.$

Case 4: $T_r = \dfrac{4(1-\alpha)(c_3-c_3^r)(a+c_2-c_3-c_3^r)}{3(a+c_2-2c_3^r)}.$

Cases 6 and 7: $T_r = \dfrac{(1-\alpha)(c_3-c_3^r)(6a-10\alpha c_1+12c_2+(5\alpha-9)(c_3-c_3^r))}{4(a-\alpha c_1+2c_2+(\alpha-3)c_3^r)}.$

7. Conclusion

We have presented and analyzed a framework to study contingent dual sourcing strategy (CDSS) and sole sourcing strategy (SSS) under competition and supply disruption. We find that the realized supply state of an unreliable supplier and the competitor's time to place an order are critical to the profits of a buyer that operates under supply disruption. Since a buyer cannot control the supply disruption, we propose that he orders at a strategic time that effectively mitigates the negative effects of the supply disruption on his profit. Various managerial insights from the analysis, profit comparisons, computations and study of extensions are summarized below:

- Even though CDSS has a cost advantage over SSS, it does not necessarily dominate SSS. The cost advantage of CDSS depends also on how reliable the cheaper, unreliable supplier is. That is, when his reliability level is high, the cost advantage can be significant, making CDSS a better approach. On the other hand, when the reliability is low, SSS can be a superior strategy.
- It is interesting as well as surprising that for a firm using either sourcing strategy, the maximum profit is in the case when he places the order before his competitor (who adopts the other strategy) does.
- Conventional sourcing predicts that the expected total market output in a monopoly should increase with the reliability level of the supplier. However, there is a scenario (Case 1) in which the expected total competitive market output decreases as the reliability level of the supplier increases.
- In equilibrium, asymmetric firms with different sourcing costs may choose different sourcing strategies depending on the reliability level of the unreliable supplier.
- In equilibrium, symmetric firms choose the same sourcing strategy. Specifically, when the reliability of the unreliable supplier is high and his costs are sufficiently low, the firms choose CDSS, otherwise they choose SSS.
- When CDSS is adopted with a capacity reservation with the reliable supplier, the optimal capacity to reserve is equal to his emergency order quantity.
- With capacity reservation, we derive the thresholds for per unit capacity reservation cost above which the CDSS firm does not reserve capacity.

There are possible future extensions of our research that are worth considering. One is a study of the competitive buying behavior with CDSS and SSS when suppliers have capacity limits. As a result, the buyers may not be able to order up to the level that they could without the capacity limits. This would mean that having the suppliers with limited capacities will have implications on the strategy of the buyers, and these will be worth examining. A more detailed study of endogenous sourcing than that carried out in Section 6.1 would reveal how the cost asymmetry and the reliability level of the unreliable suppliers interact. Specifically, what is the threshold level of the unreliability for the choice of different strategies given the sourcing costs.

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Appendix: Proofs and Supplementary Explanations

Proof of Proposition 1 The derivatives of Q_1^{1*} with respect to α, c_1, c_2 and c_3, respectively, are $\frac{\partial Q_1^{1*}}{\partial \alpha} = \frac{a - 2c_1 - 2c_2 + 3c_3}{2(\alpha + 2)^2} > 0$, $\frac{\partial Q_1^{1*}}{\partial c_1} = -\frac{\alpha + 1}{\alpha + 2} < 0$, $\frac{\partial Q_1^{1*}}{\partial c_2} = \frac{1}{(\alpha + 2)} > 0$, and $\frac{\partial Q_1^{1*}}{\partial c_3} = -\frac{1 - \alpha}{\alpha + 2} < 0$. Similarly, we can prove the results for Q_2^{1*} and Q_e^{1*}. □

Proof of Proposition 2 $\frac{\partial E\left(S^1\right)}{\partial \alpha} = \frac{-a - \left(\alpha^2 + 4\alpha + 2\right)c_1 + 2c_2 + \left(\alpha^2 + 4\alpha + 1\right)c_3}{2(\alpha + 2)^2}$. So, if $2c_2 + \left(\alpha^2 + 4\alpha + 1\right)c_3 > a + \left(\alpha^2 + 4\alpha + 2\right)c_1$, we have $\frac{\partial E\left(S^1\right)}{\partial \alpha} > 0$. It is easy to verify that $\frac{\partial E\left(S^1\right)}{\partial c_1} < 0$, $\frac{\partial E\left(S^1\right)}{\partial c_2} < 0$, and $\frac{\partial E\left(S^1\right)}{\partial c_3} < 0$. □

Proof of Proposition 3

$$\frac{\partial E\left(\mathrm{S}^2\right)}{\partial\alpha} = \frac{1}{\left(3\alpha+4+\alpha^2\right)^2}(6c_2 - c_3 + a(-1+\alpha(2+\alpha)) - c_1(1+\alpha)(4+\alpha(12+\alpha(5+\alpha))) + \alpha(4c_2 + c_3(10+\alpha(16+\alpha(6+\alpha))))).$$

If $6c_2 - c_3 + a(-1+\alpha(2+\alpha)) - c_1(1+\alpha)(4+\alpha(12+\alpha(5+\alpha))) + \alpha(4c_2 + c_3(10+\alpha(16+\alpha(6+\alpha)))) > 0$, then $\frac{\partial E\left(\mathrm{S}^2\right)}{\partial\alpha} > 0$. It is easy to verify that $\frac{\partial E\left(\mathrm{S}^2\right)}{\partial c_1} < 0, \frac{\partial E\left(\mathrm{S}^2\right)}{\partial c_2} < 0$ and $\frac{\partial E\left(\mathrm{S}^2\right)}{\partial c_3} < 0$. □

Proof of Proposition 4 $\frac{\partial E\left(\mathrm{S}^3\right)}{\partial\alpha} = \frac{c_3 - c_1}{2} > 0$. Therefore, the expected total market output $E\left(\mathrm{S}^3\right)$ is increasing in α. Similarly, we can prove the other results for $E\left(\mathrm{S}^3\right)$ and $E\left(p^3\right)$. □

Proof of Proposition 5 $\frac{\partial E\left(\mathrm{S}^4\right)}{\partial\alpha} = \frac{a - 6c_1 + c_2 + 4c_3}{12} > 0$. Therefore, the expected total market output $E\left(\mathrm{S}^3\right)$ is increasing in α. Similarly, we can prove other results for $E\left(\mathrm{S}^4\right)$ and $E\left(p^4\right)$. □

Proof of Proposition 6 Proposition 6 is easily derived from the first derivative of $E\left(\mathrm{S}^5\right)$ and $E\left(p^5\right)$ with respect to α, c_1, c_2, and c_3, respectively. □

Proof of Proposition 7 We can prove this result by taking the first derivatives of $E\left(\mathrm{S}^6\right)$ and $E(p^6)$ with respect to α, c_1, c_2, and c_3, respectively. □

Proof of Proposition 8 By straightforward comparison of the profit expressions obtained in Section 3. □

Proof of Proposition 9 We have from the expected profits calculated before and $c_2 = c_3$ that $E\left(\Pi_{\mathrm{C}}^5\right) > \alpha\frac{(a-c_2)^2}{8} + (1-\alpha)\frac{(a-c_2)^2}{16}$ and $E\left(\Pi_{\mathrm{S}}^5\right) < \alpha\frac{(a-c_2)^2}{16} + (1-\alpha)\frac{(a-c_2)^2}{8}$. Substituting $\alpha = 0.5$ in both inequalities we have $E\left(\Pi_{\mathrm{C}}^5\right) > \frac{3(a-c_2)^2}{32}$ and $E\left(\Pi_{\mathrm{S}}^5\right) < \frac{3(a-c_2)^2}{32}$. Therefore, $E\left(\Pi_{\mathrm{C}}^5\right) > E\left(\Pi_{\mathrm{S}}^5\right)$, which completes the proposition. This means that when Supplier U is sufficiently reliable, i.e. $\alpha \geq 0.5$, C has a higher expected profit than S. □

Proof of Proposition 10 Let the capacity reserved by C be $K^*(c_r)$, and the emergency order quantity at c_r be $Q_e^{k*}(c_r)$. Correspondingly, there are three cases – (a) $K^*(c_r) > Q_e^{k*}(c_r)$; (b) $K^*(c_r) < Q_e^{k*}(c_r)$; and (c) $K^*(c_r) = Q_e^{k*}(c_r)$. In Case (a); the reserved capacity is higher than the emergency order quantity. C saves $c_r(K^*(c_r) - Q_e^{k*}(c_r))$ by reserving a capacity of $Q_e^{k*}(c_r)$. Therefore, it is not optimal for C to choose $K^*(c_r) > Q_e^{k*}(c_r)$. On the other hand, in Case (b) the reserved capacity $K^*(c_r)$ is less than $Q_e^{k*}(c_r)$. Therefore, C cannot fulfil the optimal emergency order. Therefore, it is not optimal for C to choose $K^*(c_r) < Q_e^{k*}(c_r)$. Therefore, $K^*(c_r) = Q_e^{k*}(c_r)$ is optimal for C. From Section 3.1, we see that Q_e^{k*} decreases in c_3^r and c_r is a sunk cost to decide Q_e^{k*}. □