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A WTP-Choice Model: Empirical Validation, Competitive and Centralized Pricing

Varun Gupta
Black School of Business, Penn State Erie, The Behrend College, Erie, Pennsylvania 16563, USA, vxg15@psu.edu

Metin Çakanyıldırım
Naveen Jindal School of Management, University of Texas at Dallas, Richardson, Texas 75080, USA, metin@utdallas.edu

Willingness To Pay (WTP) of customers plays an anchoring role in pricing. This study proposes a new choice model based on WTP, incorporating sequential decision making, where the products with positive utility of purchase are considered in the order of customer preference. We compare WTP-choice model with the commonly used (multinomial) Logit model with respect to the underlying choice process, information requirement, and independence of irrelevant alternatives. Using WTP-choice model, we find and compare equilibrium and centrally optimal prices and profits without considering inventory availability. In addition, we compare equilibrium prices and profits in two contexts: without considering inventory availability and under lost sales. One of the interesting results with WTP-choice model is the "loose coupling" of retailers in competition; prices are not coupled but profits are. That is, each retailer should charge the monopoly price as the collection of these prices constitute an equilibrium but each retailer's profit depends on other retailers' prices. Loose coupling fails with dependence of WTPs or dependence of preference on prices. Also, we show that competition among retailers facing dependent WTPs can cause price cycles under some conditions. We consider real-life data on sales of yogurt, ketchup, candy melt, and tuna, and check if a version of WTP-choice model (with uniform, triangle, or shifted exponential WTP distribution), standard or mixed Logit model fits better and predicts the sales better. These empirical tests establish that WTP-choice model compares well and should be considered as a legitimate alternative to Logit models for studying pricing for products with low price and high frequency of purchase.

Key words: willingness-to-pay; choice models; competitive pricing; centralized pricing; lost sales; estimation
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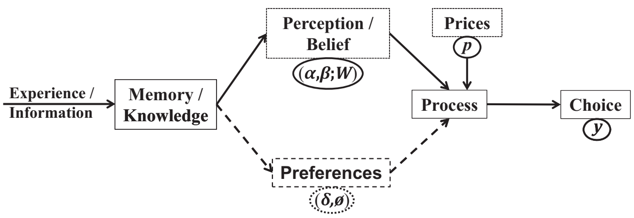
1. Introduction

Willingness To Pay (WTP) is the maximum amount a customer would be willing to pay in order to receive a product and it plays a central role in the selection of a product from several choices. The primary aim of this study is to propose a discrete choice model based on WTPs and to apply the model to competitive and centralized pricing. In competitive pricing, each retailer sells a product and determines its price to compete with the other retailers selling similar products. In centralized pricing, a retailer sells several products and determines their prices to maximize the profit. The proposed WTP-based choice model is used to study competitive pricing, first by focusing on dependencies among WTPs and customer preferences and by ignoring inventory consideration, and then under lost sales that alter customer choices. The same model is also used in centralized pricing, and centrally optimal prices are compared with the equilibrium in competitive pricing. Another aim is to check the efficacy of the WTP-choice model by comparing it with the commonly used (multinomial) Logit model and mixed Logit model in terms of the log-likelihood values as well as the accuracy of choice estimates. Comparisons involve real-life data on candy melts, yogurt, ketchup, and tuna sold by different retailers (firms) in different markets.

The classical approach to customer choices is through Logit models (e.g., McFadden 1980). Logit model and its extensions have so far been the preferred model (Chandukala et al. 2008, Schroeder 2010, pp.78–85). The customer choice literature is growing with the exploration of the process of forming perceptions and beliefs in different practical contexts. Figure 1 shows the choice process for a customer given his experiences and information. The perceptions and preferences of a customer shaped from his memories and knowledge of products as well as the prices lead to the product choice. As opposed to Figure 1, McFadden (2001) connects a customer's memory/knowledge to the decision process with a single path by combining his perception/belief and preferences. In this study, we separate a customer's perception/belief and preferences using the lower path in Figure 1. Through this separation, we

Figure 1 A Choice Process with Its Inputs and Output



explicitly represent customer preferences. Therefore, in line with the direct utility approach (Chintagunta and Nair 2011) to customer choice modelling, WTP-choice model directly incorporates WTPs and customer preferences, and provides direct insights into customer behavior.

Willingness To Pay-choice model is motivated by our interaction with retail executives, who suspect that sales for their products are driven not only by the prices but also by the customer preferences (e.g., habits and brand recognition of the products) based on customers' past experiences. They typically run customer surveys to find out the customer preferences. The premise behind modelling preferences includes capturing a customer's established habits, routine, convenience of shopping in certain patterns or sequences, and relative magnitude of search/transportation costs for retailers/products. Dillon et al. (2013) studied two Chicago-based grocery stores: Jewel and Dominick's. They found out that respectively 53% and 41% of shoppers are likely to first visit Jewel and Dominick's, which shows that shoppers exhibit habits. These habits are collectively called preferences and can be independent of product prices. Preferences can also be deployed as measures of customer loyalty (Bijmolt et al. 2010). One can argue that the parameters of Logit model capture choice probabilities, which may *indirectly* be interpreted as (revealed) preferences. Without resorting to such an interpretation, we start with (stated) preferences *directly* incorporated into WTP-choice model.

Product prices and utility (quality) vary across retail stores and over time. Most choice models such as Logit model assume that the customers are aware of the prices and utilities for all products. In reality, customers are unlikely to maintain such extensive information especially for high-purchase frequency items, even if they are willing to spend the cognitive effort for objectively processing this information, which may lead to a state of paralysis-by-analysis. Instead, each customer gathers knowledge about some, but perhaps not all, products to make a choice, e.g., Seiler (2010) maintains that customers infrequently check product prices and quality at a few retail stores. Preferences help a customer make a choice before checking all products (Carlson et al. 2009). Hauser and Wernerfelt (1990) showed that customers have *consideration sets* whose size can be as low as 2 depending on product categories. Hauser (2014) reviewed heuristic rules for first forming a consideration set and then choosing a product from this set. WTP-choice model explicitly models customers' consideration set, as well as the consideration set heterogeneity among the customers. It uses preferences to explicitly rank the products within a particular consideration set. It hence captures the sequential search for a product, which usually ends before considering all products.

As decision rules, customers use two-stage decision process and threshold screening to simplify complicated decisions (Gilbride and Allenby 2004). A customer may be satisfied with a reasonable product and stop searching for better products (Stüttgen et al. 2012) when prices are unavailable or costly to find out. Such a customer does not necessarily maximize his surplus—difference between the utility obtained from a product and its price. Bounded rationality (Gigerenzer and Selten 2002) of customers can be used to explain why customers do not always maximize their surplus and instead are satisfied with just non-negative (feasible) surplus. Even extremely rational customers may not care about tedious surplus maximization when buying low-value items, whereas they can be quite meticulous when buying high-value items. It is better to assess the appropriateness of a customer's search for maximum vs. feasible surplus after specifying the product and its value. Hence, the domain of choice models is very broad and can accommodate new models, especially those that are simple, based on customer preferences and a feasible surplus criterion.

In Logit model, customer n chooses the product i from M products at a price p^i with the probability

$$\lambda_n^i = \exp(\alpha_i + \beta p_n^i)\Big[\sum_{j=0}^{M} \exp(\alpha_j + \beta p_n^j)\Big]^{-1}. \quad (1)$$

The choice of $i = 0$ indicates *no-purchase* with $p_n^0 = 0$. The term $\exp(\alpha_i + \beta p_n^i)$ is the attractiveness of option i and its increase/decrease in price p_n^i is governed by parameters $\alpha_0, \alpha_1, \ldots, \alpha_M$ and β (Cameron and Trivedi 2005, pp. 491–495). Each customer maximizes the surplus and each utility has a double-exponential distribution (Talluri and Van Ryzin 2004, p. 306). Recently, Farias et al. (2013) and Jagabathula and Rusmevichientong (2013) propose non-parametric choice models consistent with Logit model and then predict revenues respectively for an automaker and a television retailer.

To better appreciate the differences between WTP-choice model and Logit model, we provide an example of a customer who considers buying organic yogurt or regular yogurt, and prefers organic yogurt. In real-life, the choice of this organic yogurt preferring customer may not be affected by the price of regular yogurt. This is because such a customer buys the organic yogurt if it is affordable. Basing the choice only on the affordability of the preferred product corresponds to the satisficing criterion of WTP-choice model. On the contrary, this choice, if captured by Logit model, is affected by the price of regular yogurt due to surplus maximization over organic and regular yogurts.

Many items at retailers experience stockouts (DeHoratius et al. 2008), so the impact of a stockout on the choice process is important to investigate. Choice models such as Logit model or location choice model cannot explicitly capture the effects of a stockout on the customer behavior (Gaur and Honhon 2006). To capture these, Musalem et al. (2010) used a modified Logit model. Mahajan and van Ryzin (2001) took stockout events into account by dynamically removing the stocked-out items from the customer's consideration set. Kök and Fisher (2007) highlighted the inability of the standard Logit model to capture the impact of stockouts on customer behavior, developed a demand rate function to capture this behavior and used it for demand estimation and assortment optimization. However, these works did not study pricing decisions.

Choice models are the building blocks of price optimization (Özer and Phillips 2012). Alptekinoğlu and Semple (2016) propose an exponomial choice model and compare price optimization results obtained using exponomial choice model with those obtained by using Logit model. Both exponomial and Logit frameworks lead to non-trivial price optimization (Li and Huh 2011). When customers do not maximize their utility as in the case of low-price items (Stüttgen et al. 2012) or when the utility distribution does not follow the double-exponential distribution as in Logit model (or the normal distribution as in Probit model), one needs a new choice model, possibly based on WTPs.

Willingness To Pay estimation has received significant attention and uses scanner or survey data (Wertenbroch and Skiera 2002). In the scanner data methods, there are buyers and non-buyers. WTP of a buyer is at least the price being offered and that of a non-buyer is less than the price being offered. Earlier studies assumed WTP to be a single price point in customer's propensity to buy, however, later studies consider it to be a range (Wang et al. 2007). WTP can be indirectly constructed by starting with a utility framework, however, estimating it directly fits the data better in general, decreasing the chances of exceedingly large estimated WTP variances (Scarpa et al. 2008). Our paper parametrically estimates WTP distributions using a likelihood criterion. It should be noted that WTP does not have to be parameterized for WTP-choice model; only in estimation and sharpening some results, we resort to parameterization.

We compare Logit model and WTP-choice model and show that competitive pricing with WTP-choice model is relatively easy to analyze and implement. In particular with WTP-choice model, the prices are "loosely coupled"; each retailer should charge monopoly prices in competition as these constitute the equilibrium, but that retailer's profit depends on prices of all the retailers. At the onset, loose coupling seems to be surprising, this however relates to *monopolistic competition*, where "each firm . . . can ignore its impact on, and hence reactions from, other firms" (Hart 1985, p. 529). Monopolistic competition in a market is due to the presence of customers who differentiate between the brands in the market but do not easily switch to another brand due to slight changes in the price of a brand. This description of customers who exhibit a friction to brand switching hints at price-independent customer preferences. However, we note that these preferences are not sufficient for loose coupling, which vanishes with dependent WTPs. Independence of WTPs and the independence of preferences from prices together drive loose coupling and can yield closed-form expressions for equilibrium prices.

We illustrate with an example that dependence of WTPs can cause *price cycles*, where prices charged by the retailers alternate within a set of prices. We provide conditions to rule out price cycles and to conclude the presence of an equilibrium with a single price for each retailer. Although price cycles are not usually considered as a solution to a pricing game in Operations Management, they are empirically observed (Noel 2008) and theoretically explained (Maskin and Tirole 1988). Dependence of WTPs in our model can provide another explanation for these cycles.

In the centralized pricing context, the optimal price of a product depends on the price of another product, which rules out separation (counterpart of loose coupling) of the pricing problem over prices. First, this implies that competitive prices are easier to solve for than centralized prices, which is an interesting observation on its own right as decentralized problems (involving competitive games) are generally more challenging. Second, the profit objective is not necessarily concave so finding centrally optimal prices require some development. Along these lines, an iterative algorithm is provided and is shown to yield the globally optimal prices for uniform and shifted exponential WTP distributions.

We extend WTP-choice model to study competitive pricing under substitutions driven by lost sales. Substitutions in our model are based on lost sales probabilities rather than lost sales events. Compared to lost sales events, lost sales probabilities are more stable in the sense that a probability is the average frequency of many events. This stability makes lost sales probabilities for customers easier to obtain and use in a choice model (Hopp and Xu 2008). By modeling lost sales, we illustrate the versatility of WTP-choice model and test the robustness of loose coupling property. Provided that WTPs are independent of each other and preferences are independent of prices, this property continues to hold.

Choice models map prices to choices through parameters that need to be estimated (Olivares et al. 2008, Akşin et al. 2013). Although the proposed WTP-choice model is non-parametric, we parameterize the WTP distribution to simplify the estimation scheme, which uses likelihood maximization. We assess the estimation and prediction efficacy of WTP-choice models in comparison with Logit models using real-life data on yogurt, ketchup, candy-melt, and tuna. The estimation and prediction results show that WTP-choice models compare well with Logit models.

This paper's contributions include introducing WTP-choice model in section 2, establishing a loose coupling property for competing retailers under independent WTPs and studying equilibrium prices and price cycles under dependent WTPs in section 3, finding centralized prices and comparing them to equilibrium prices in section 4, extending the loose coupling property to inventory models with lost sales in section 5 and empirically validating WTP-choice model in section 6.

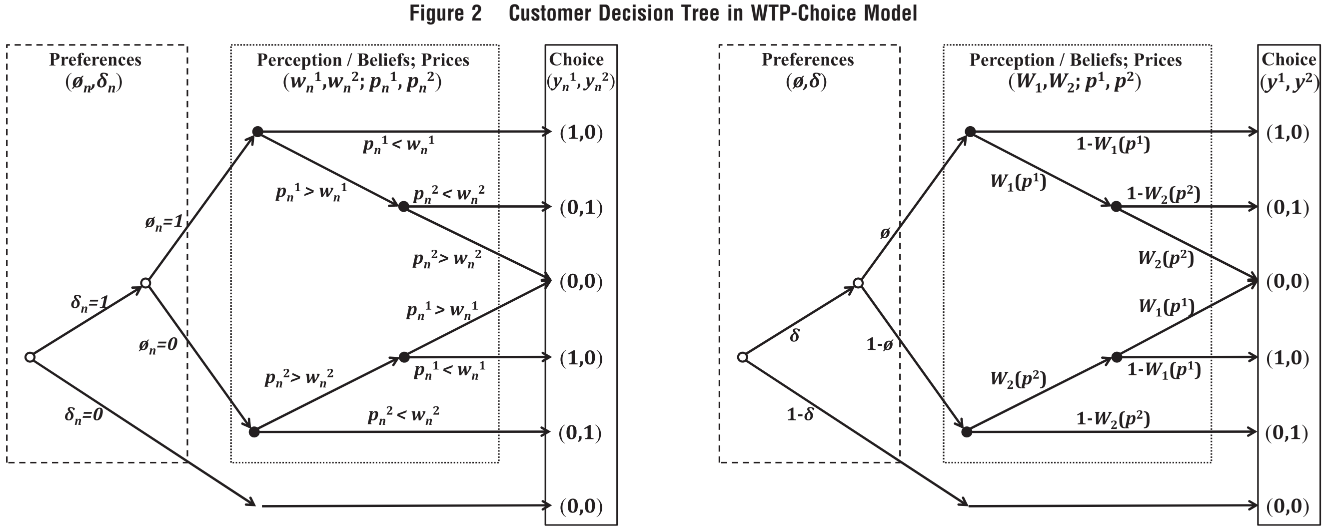
2. WTP-Choice Model

Willingness To Pay-choice model is designed to incorporate WTPs and customer preferences into the choice process and it applies to a context of M products in a market. Customer n is offered product $m \le M$ at the price p_n^m. Subsequently, customer n decides to buy ($y_n^m = 1$) product m or not ($y_n^m = 0$). In WTP-choice model, customer n has the preferences that are part of the lower path in Figure 1. The parameter δ_n captures the preference of customer n to buy a product or nothing. For example, customer n with $\delta_n = 0$ is interested in none of the products.

To explain product preferences and WTP-choice model, we first consider two products, so $M = 2$. The parameter ϕ_n denotes the preference of customer n: if the customer prefers product 1, he has $\phi_n = 1$; otherwise, $\phi_n = 0$. When $\delta_n = 0$, customer n is not in the market to buy product 1 or 2. For example, such a customer may enter a store to buy other products but does not pay attention to products 1 or 2. When $\delta_n = 1$, customer n walks in the store and checks out the prices of products 1 and 2 to decide to buy or not. This customer does not buy a product if he gets a negative surplus from each product, i.e., the price of each product is higher than his WTP for that product. Hence, customer n buys nothing, i.e., $(y_n^1, y_n^2) = (0, 0)$, when he is uninterested or prices are high relative to WTPs. These respectively correspond to the two (0, 0) choices in Figure 2a.

When $(\delta_n, \phi_n) = (1, 1)$, customer n is in the market to buy a product and prefers product 1 over product 2. If this customer's surplus from product 1 is nonnegative, he buys product 1. The customer arrives at this decision without considering product 2. If the

Figure 2 **Customer Decision Tree in WTP-Choice Model**



(a) Customer's perspective. **(b)** Firm's perspective.

surplus from product 1 is negative, then the customer considers product 2. If the surplus from product 2 is non-negative, he buys it. Otherwise, he buys nothing. The choice process for a customer with $(\delta_n, \phi_n) = (1, 0)$ is symmetric to the process described above and is shown in Figure 2a.

Given prices (p_n^1, p_n^2), preferences (δ_n, ϕ_n), and customer WTPs (w_n^1, w_n^2), choices of customer n are:

- $(y_n^1, y_n^2) = (1, 0)$: Product 1 if $[\delta_n = 1, \phi_n = 1, p_n^1 \leq w_n^1]$ or $[\delta_n = 1, \phi_n = 0, p_n^2 > w_n^2, p_n^1 \leq w_n^1]$.
- $(y_n^1, y_n^2) = (0, 1)$: Product 2 if $[\delta_n = 1, \phi_n = 0, p_n^2 \leq w_n^2]$ or $[\delta_n = 1, \phi_n = 1, p_n^1 > w_n^1, p_n^2 \leq w_n^2]$.
- $(y_n^1, y_n^2) = (0, 0)$: None if $[\delta_n = 1, p_n^1 > w_n^1, p_n^2 > w_n^2]$ or $[\delta_n = 0]$.

A firm often does not know the preferences or WTPs of each customer as it faces a population of customers. This population has preferences $\{\phi_1, \phi_2, \ldots\}$ for $\phi_n \in \{0, 1\}$, and the firm can estimate the probability ϕ that a random customer prefers product 1 over 2. Similar to ϕ, we can use δ for the probability that a random customer is interested in a product. Unlike (δ, ϕ), WTP random variables W^1 and W^2 are not binary variables, so the probability associated with them can be represented by a cumulative probability distribution $P(W^1 \leq p^1, W^2 \leq p^2)$ for $p^1, p^2 \geq 0$. We assume independence of WTPs in this study except for section 3.2–3.3, so we obtain $P(W^1 \leq p^1, W^2 \leq p^2) = P(W^1 \leq p^1)P(W^2 \leq p^2)$. We often use cumulative probability distribution W_i of W^i, i.e., $W_i(p^i) := P(W^i \leq p^i)$. Figure 2b uses probabilities (δ, ϕ, W_1, W_2) to present a single random customer's choice from the firm's perspective. In general, these probabilities model two contexts: (i) all customers have the same WTP but this WTP is unknown to the firm, (ii) all customers have different WTPs that come from the same distribution.

Given prices, preferences, and independent WTPs, the choice probabilities for customer n are:

$$\rho_n^1 := P(y_n^1 = 1, y_n^2 = 0) = \delta(1 - W_1(p_n^1))\{(1-\phi)W_2(p_n^2) + \phi\}, \quad (2)$$

$$\rho_n^2 := P(y_n^1 = 0, y_n^2 = 1) = \delta(1 - W_2(p_n^2))\{\phi W_1(p_n^1) + (1-\phi)\}, \quad (3)$$

and $\rho_n^0 = \delta W_1(p_n^1)W_2(p_n^2) + 1 - \delta = 1 - \rho_n^1 - \rho_n^2$. The first term on the right-hand side of Equation (2) is the probability that the customer is interested in a product. The second term in the parentheses is the probability that the customer is willing to pay at least the price of product 1. The third term in brackets expresses the sum of the probabilities that the customer prefers product 2 but finds it too expensive and that the customer prefers product 1. A similar interpretation can be given for the probability ρ_n^2.

In the M-product version of WTP-choice model, we assume that customers have a collection of ordered consideration sets of $\mathfrak{L}_i$ for $i = 1, \ldots, S$ and each set has size $L \leq M$. Each product belongs to at least one of the consideration sets. The probability that a customer has the consideration set $\mathfrak{L}_i$ is ϕ_i and $\sum_{i=1}^S \phi_i = 1$. We use $\mathfrak{L}_i^{<m}$ to denote the set of products in $\mathfrak{L}_i$ that are preferred to product m. The choice probability ρ_n^m is given by

$$\rho_n^m := \delta(1 - W_m(p_n^m)) \sum_{i=1}^S \phi_i \mathbb{1}_{m \in \mathfrak{L}_i} \prod_{j \in \mathfrak{L}_i^{<m}} W_j(p_n^j) \quad (4)$$

and $\rho_n^0 = 1 - \sum_m \rho_n^m$. Here, $\mathbb{1}_{\mathcal{A}}$ represents the indicator function which is 1 when $\mathcal{A}$ holds and 0 otherwise. In WTP-choice model, each customer checks the retailers in its consideration set $\mathfrak{L}_i$ one by one without forming estimates about the prices at the retailers to be checked. This is in line with bounded rationality of customers, especially when buying low-price items, motivated earlier. Although we briefly use customer index as subscript of ϕ above to explain probability ϕ, the subscript of ϕ in the remainder is always a consideration set index or a product index when $M = 2$. Setting $\phi = \phi_1 = 1 - \phi_2$ is also a convention adopted for $M = 2$ in the remainder.

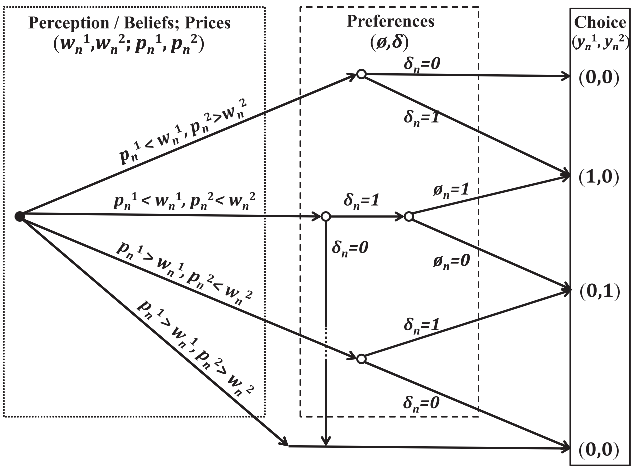
The choice probabilities in Equations (2–3) are obtained with $(M, L) = (2, 2)$ and $\mathfrak{L}_1 = \{1, 2\}$, $\mathfrak{L}_2 = \{2, 1\}$. We also illustrate an example with $(M, L) = (3, 2)$, i.e., a population of customers choose among three products and each customer's consideration set has size 2. All possible considerations sets are $\mathfrak{L}_1 = \{1, 2\}$, $\mathfrak{L}_2 = \{1, 3\}$, $\mathfrak{L}_3 = \{2, 1\}$, $\mathfrak{L}_4 = \{3, 1\}$, $\mathfrak{L}_5 = \{2, 3\}$, $\mathfrak{L}_6 = \{3, 2\}$. Therefore from Equation (4), $\rho_n^1 = \delta(1 - W_1(p_n^1))\{\phi_1 + \phi_2 + \phi_3 W_2(p_n^2) + \phi_4 W_3(p_n^3)\}$.

When all customers are offered the same price $p^{\mathcal{M}} := \{p^1, p^2, \ldots, p^M\}$, the choice probabilities in Equations (1), (2–3), or (4) do not depend on the customer index n. Logit and WTP-choice models take the same price and sales data and output choice probabilities. Hence, they can be used in the same context and their comparison in the following three aspects is important.

2.1. Sequences

A natural question is whether the sequence of events (learning prices and forming preferences) affect the outcome of the choice process. WTP-choice process above is conceived by assuming that customers learn

Figure 3 Alternative Decision Tree when Prices are Learnt First



prices in the last stage after they assess their preferences as in Figure 2a. Figure 3 on the contrary shows customers that learn prices first as in an e-commerce context. We can check that the same events lead to the same choices in both Figures 2a and 3. Hence, WTP-choice model is robust with respect to the sequence of forming preferences and learning prices.

Logit model is not based on any explicit product sequence, as it assumes that a customer decides after collecting prices and assessing utilities for all products. Given choice probabilities, the probability for each sequence of considering products can be induced (Luce 1977). Although this gives a probability for such a sequence, it always requires consideration of all the products as the customers decide to purchase *at-once* after reaching the end of the sequence. In the nested Logit models (Danaher and Dagger 2012), there is a natural product hierarchy: product groups at the top level and products at the bottom level. To illustrate, we can consider the choice of a dessert (ice-cream, frozen yogurt, cakes) as the product group and the choice of flavor (mint ice-cream, strawberry yogurt, chocolate cake) as the product. Nevertheless, the customer in a nested Logit model also requires all of the attributes of products to decide at-once. Therefore, regular and nested Logit models assume at-once decision making. This is a fundamental difference between Logit and WTP-choice models.

2.2. Information Requirement

Logit model requires customers who are informed with prices $p^{\mathcal{M}}$ to decide at-once, whereas WTP-choice model envisions customers who sequentially acquire price information only for the product under consideration. WTP-choice model on average requires less price information than Logit model, so the former model is more appropriate when the customers' search cost is high relative to the price (Cachon et al. 2005). For example, customer n can traverse the top path in Figure 2a to end up with the choice (1, 0) without requiring p_n^2 or w_n^2.

2.3. Independence of Irrelevant Alternatives (IIA) Property

Logit model has the IIA property, i.e., the relative odds of choice between two alternatives is not affected by the addition of another alternative. IIA property of Logit model has been criticized in the literature (Luce 1977, Train 2009). Mixed or nested version of Logit model or the exponomial choice model (Alptekinoğlu and Semple 2016) does not have the IIA property, nor does WTP-choice model. First, we calculate the relative odds in WTP-choice model with two options: *buy* product 1 and *not buy*. This ratio is $\rho^1(p^1)/\rho^0(p^1) = \delta(1 - W_1(p^1))/\delta W_1(p^1) + 1 - \delta$. Adding another alternative—product 2 sold at price p^2 and its WTP distribution W_2—we get another ratio $\rho^1(p^1, p^2)/\rho^0(p^1, p^2) = \delta(1 - W_1(p^1))\ \{(1 - \phi) \times W_2(p^2) + \phi\}/\{\delta W_1(p^1)W_2(p^2) + 1 - \delta\}$. These ratios are not always identical, so addition of an alternative changes the relative odds.

3. WTP-Choice Model in Competitive Pricing

Retailers often require a choice model that acts as an input for maximizing their profit. We consider a competitive pricing context, where each retailer owns a product and determines its price. In particular, a retailer decides the price p^i of product i in expectation of the prices $p^{\mathcal{M}\setminus i} := p^{\mathcal{M}} \setminus p^i$ of the competing products. Equilibrium prices are studied under the following four settings: independent WTPs, dependent and continuous-valued WTPs, dependent and discrete-valued WTPs, and price-dependent preference. The last three settings yield interesting insights, but the first setting of independent WTPs goes a long way to predict the sales in section 6.

3.1. Independent Willingness-to-Pays

Assuming that retailers do not collude among each other, the objective for pricing product i to maximize the profit of retailer i from a single customer is

$$\Pi_i(p^{\mathcal{M}}) = (p^i - c_i)\rho^i(p^1, p^2, \ldots, p^M)$$
$$= (p^i - c_i)(1 - W_i(p^i))\left[\delta \sum_{j=1}^{S} \phi_j \mathbb{1}_{i \in \mathfrak{L}_j} \prod_{k \in \mathfrak{L}_j^{<i}} W_k(p^k)\right], \tag{5}$$

where c_i is the cost per unit for retailer i. Since the terms above in the square brackets are constant in p^i, the response price $\mathcal{P}_i(p^{\mathcal{M}\setminus i})$, i.e., the optimal price for given $p^{\mathcal{M}\setminus i}$, of retailer i is the maximizer of $(p^i - c_i)(1 - W_i(p^i))$. In other words, the response price $\mathcal{P}_i(p^{\mathcal{M}\setminus i})$ to competitor prices $p^{\mathcal{M}\setminus i}$ is independent of $p^{\mathcal{M}\setminus i}$ when the WTPs are independent. Hence, the price p^i can be optimized without knowing competitor prices, the WTP distributions, or preferences for the competitor products. The response price p^i satisfies

$$\frac{p^i}{p^i - c_i} = p^i\left[\frac{w_i(p^i)}{1 - W_i(p^i)}\right] =: \Lambda_i(p^i), \tag{6}$$

where $w_i(p) = dW_i(p)/dp$, the term inside square brackets is the failure rate function of the WTP distribution $W_i(p^i)$ and $\Lambda_i(p^i)$ is the generalized failure rate function of the same distribution. From Equation (6), a retailer's response price does not depend on the prices of other retailers, which is formally stated as the loose coupling property in Theorem 1(a) below.

Failure rate functions are well studied for many distributions, e.g., uniform, gamma, Weibull (with shape parameter >1), truncated normal and modified extreme value distributions have increasing generalized failure rates, so the response price is unique. Besides providing such a uniqueness property, the optimality equation $\Lambda_i(p^i) = p^i/(p^i - c_i)$ is simple, especially because $\Lambda_i(p^i)$ can be looked up from literature (Birolini 2010, pp. 433–446). We assume that WTPs have increasing failure rate unless otherwise is said. This implies that $\Lambda_i(p^i)$ is increasing. The increasing failure rate assumption is used to obtain unimodality of profit functions and uniqueness of their maximizers as in Theorem 1(b) below. We suppose that WTPs are distributed over intervals $[a_i, b_i]$ for $a_i, b_i \geq 0$ and allow for $a_i = 0$ and $b_i \to \infty$. If $a_i < c_i$, we can ignore WTPs lower than c_i and consider the rest, whose distribution is $W_i(\cdot)/(1 - W_i(c_i))$. Hence, we can assume $a_i \geq c_i$ without loss of generality. Then, $W_i(a_i) = 0$ and $W_i(b_i) = 1$, so a customer buys when $p^i \leq a_i$ and nobody buys when $p^i \geq b_i$. The profit $\Pi_i(p^{\mathcal{M}})$ is strictly increasing in p^i as $\rho_i(p^{\mathcal{M}})$ is constant for $p^i \leq a_i$ and it is constant at zero for $p^i > b_i$, so it is not maximized by $p^i < a_i$ or $p^i > b_i$.

Theorem 1 (Loose coupling).

(a) *Unlike the profit function, the price response function of a retailer is independent of costs, prices, WTPs, and preferences of other retailers, so equilibrium prices are monopolistic prices.*

(b) *The profit Π_i is unimodal in p^i. It is either maximized at the end points of $[a_i, b_i]$ or at the unique root of $\Lambda_i(p^i) = p^i/(p^i - c_i)$.*

As examples for WTP distribution $W_i(p)$, we consider uniform (□) and shifted exponential (⌞). $W_i(p)$, $\Lambda_i(p)$, and the equilibrium prices p^{ie} are given in Table 1. Note that □ and ⌞ distributions have increasing generalized failure rates, consequently we have a unique best response price.

For Logit model, there is not a simple expression for the response price. The objective $(p^i - c_i)\lambda^i(p^{\mathcal{M}})$ yields an implicit equation, where both sides depend on p^i and the right-hand side depends on also $p^{\mathcal{M}\setminus i}$:

$$p^i = c_i - [\beta\{1 - \lambda^i(p^i, p^{\mathcal{M}\setminus i})\}]^{-1}. \tag{7}$$

Comparison of Equations (6) and (7) shows the simplicity of pricing with WTP-choice model. Standard logit model applies only to the case of independent utilities without inventory consideration, so our next comparison of Logit and WTP-choice model takes place in section 6.

Loose coupling in Theorem 1 is an important property for various reasons. First, it is a non-parametric result that holds with any WTP distribution. Second, a retailer in competition with others does not need to know anything about the costs, prices, or WTPs for the products sold by others. This tremendously decreases the amount of information required by a single retailer to decide on his price and greatly facilitates the implementation of WTP-choice model. In practice, the highest hurdle to set up an equilibrium formulation is the exact and timely information required by a retailer; this hurdle is significantly lowered by loose coupling. Third, loose coupling simplifies computations as it suffices for the equilibrium price p to solve $p/(p - c) = \Lambda(p)$, which is an equation of a single variable, or to plot $p/(p - c)$ and $\Lambda(p)$ to find their intersection. Because of these reasons, it is worthwhile to check if loose coupling remains valid in various contexts.

Loose coupling is a striking result that relates to the monopolistic competition. Monopolistic competition occurs when prices change slightly and customers resist to switching from one product to another. This resistance is captured more by price-independent preferences than their dependent counterparts. When the preferences are dependent on prices, one may expect loose coupling to fail, which we show in

Table 1 Prices with Different Willingness To Pay Distributions

Distribution	$W_i(p)$	$\Lambda_i(p)$	p^{ie}
Uniform □ (a_i, b_i)	$(p - a_i)/(b_i - a_i)$	$p/(b_i - p)$	$\max\{a_i, (b_i + c_i)/2\}$
Shifted exponential ⌞ $(a_i, \infty; \tau_i)$	$1 - \exp(-\tau_i(p - a_i))$	$p\tau_i$	$\max\{a_i, 1/\tau_i + c_i\}$

section 3.4. However, it is not clear without a rigorous analysis if the loose coupling property holds in the cases of dependent WTPs. We analyze this in a duopoly—a market with two firms, each of which sells a product. For example, Fedex with UPS and AutoZone with O'Reilly Automotive constitute a duopoly, respectively, in *Air Freight* and in *Automotive Retail*. A firm in a market with multiple firms usually benchmarks itself against another firm that often leads in terms of revenue. For example, Target is a *General Merchandise Store*, a category led by Wal-Mart in revenue. So, Target can be paired with Wal-Mart to have a duopoly, despite the presence of smaller competing firms. Some other notable examples of duopolies are Pepsi and Coca-Cola, and Kleenex and Puffs, so duopolies are not uncommon and imply $M = L = 2$ in our notation. In case of oligopolies, a firm can view other firms as a single fictional aggregate firm and decide on its own price in a duopoly including itself and this fictional firm. To the extent that WTPs can be aggregated accurately, this approximation will work well.

3.2. Dependent and Continuous-Valued Willingness-to-Pays

To extend WTP-choice model to dependent WTPs, we start with the objective of pricing product i

$$\Pi_i(p^i, p^{-i}) = (p^i - c_i)\{\phi_i \mathrm{P}(W^i \geq p^i) + \phi_{-i}\mathrm{P}(W^i \geq p^i, W^{-i} \leq p^{-i})\}, \qquad (8)$$

where W^i is the WTP random variable for product i. Note that W_i is the distribution of W^i, so we use the subscript i for the distribution and superscript i for the random variable. In Equation (8), index $-i$ denotes the retailer other than retailer i when there are two retailers. When $i = 1$ for example, ϕ_i and ϕ_{-i} are the probabilities that a customer respectively prefers product 1 and 2. In the remaining analytical parts of the paper, we set $\delta = 1$, which, otherwise, can only scale down the profit without affecting the equilibrium or optimal prices.

We consider two examples: identical WTPs and identically distributed WTPs. We determine the equilibrium prices to identify if these prices inherit dependence (coupling) from WTPs.

3.2.1. Identical WTPs. Products which are ideal substitutes or very similar can have identical WTPs. For such WTPs, we assume uniform distribution, so $W^1 = W^2 \sim W = \square[a, b]$. Therefore, $\mathrm{P}(W^i \leq p) = (p - a)/(b - a)$ if $a \leq p \leq b$. $\mathrm{P}(W^1 \geq p^1, W^2 \leq p^2) = \mathrm{P}(p^1 \leq W \leq p^2) = (p^2 - p^1)/(b - a)$ if $a \leq p^1 < p^2 \leq b$.

When retailers set identical prices, i.e., $p^1 = p^2$, $\mathrm{P}(W^1 \geq p^1, W^2 \leq p^2) = 0$, the equilibrium prices are monopolistic prices, i.e., $p^i = (b + c_i)/2$. This is a symmetric equilibrium, with identical prices, $(p^{1e}, p^{2e}) = ((b + c)/2, (b + c)/2)$ when retailers have identical costs $c_1 = c_2 = c$.

When retailers charge different prices, say $p^1 < p^2$, we have structurally different profit maximization problems for retailers 1 and 2.

$$\text{Retailer 1:} \quad \max_p (p - c_1)\{\phi(b - p)/(b - a) + (1 - \phi)(p^2 - p)/(b - a)\} \quad \text{if } p < p^2,$$

$$\text{Retailer 2:} \quad \max_p (p - c_2)\{(1 - \phi)(b - p)/(b - a)\} \quad \text{if } p < p^1,$$

The best response price $\mathcal{P}$ for a retailer depends on the price of the other retailer, so loose coupling does not hold; $\mathcal{P}_1(p^2) = (\phi b + (1 - \phi)p^2 + c_1)/2$ and $\mathcal{P}_2(p^1) = (c_2 + b)/2$. The equilibrium is $(p^{1e}, p^{2e}) = (((1 + \phi)b + (1 - \phi)c_2 + 2c_1)/4, (c_2 + b)/2)$ if $p^{1e} < p^{2e}$. Similarly for $p^{1e} > p^{2e}$, $(p^{1e}, p^{2e}) = ((c_1 + b)/2, ((2 - \phi)b + \phi c_1 + 2c_2)/4)$. When $2c_1 - (1 - \phi)b < (1 + \phi)c_2$ we have $p^{1e} < p^{2e}$; when $(2 - \phi)c_1 + \phi b > 2c_2$ we have $p^{1e} > p^{2e}$. As these conditions are not mutually exclusive, it is possible to have both equilibria. When retailers are identical, i.e., $c_1 = c_2 = c$, $\phi = 1 - \phi = 0.5$, both conditions are satisfied on account of $c < b$. Then, we have two non-symmetric equilibria. This is very interesting as retailers charge different equilibrium prices $(p^{1e}, p^{2e}) = ((3b + 5c)/8, (c + b)/2)$ and $(p^{1e}, p^{2e}) = ((c + b)/2, (3b + 5c)/8)$, even if they are identical. The prices at these equilibria are mirror images of each other with respect to the $p^1 = p^2$ line.

3.2.2. Identically distributed WTPs. We consider $W \sim \square[0, b]$, $W^1 \sim W + \square[0, \epsilon]$ and $W^2 \sim W + \square[0, \epsilon]$ for $0 < \varepsilon < b - c$. The same W is a part of both W_1, and W_2, while realizations of $\square[0, \varepsilon]$ can be different.

Lemma 1 (Equilibrium with Identically distributed WTPs).

(a) For $\epsilon \leq p^1, p^2 \leq b$, *we have* $\mathrm{P}(W^i \geq p^i) = (\epsilon + 2(b - p^i))/(2b)$ *and*

$$\mathrm{P}(W^1 \geq p^1, W^2 \leq p^2) = \begin{cases} 0 & \text{if } p^2 \leq p^1 - \epsilon, \\ (p^2 - p^1 + \epsilon)^3/(6b\epsilon^2) & \text{if } p^1 - \epsilon \leq p^2 \leq p^1, \\ ((p^1 - p^2)^3 + 3(p^1 - p^2)^2\epsilon - 3(p^1 - p^2)\epsilon^2 + \epsilon^3)/(6b\epsilon^2) & \text{if } p^2 - \epsilon \leq p^1 \leq p^2, \\ (p^2 - p^1)/b & \text{if } p^1 \leq p^2 - \epsilon. \end{cases}$$

(b) *For identical retailers, the only symmetric equilibrium has* $p^{1e} = p^{2e} = (6b + 9c + 4\epsilon)/15$. *There are also two non-symmetric equilibria as* $(p^{1e}, p^{2e}) = ((8b + 8c + 4\epsilon)/16, (6b + 10c + 3\epsilon)/16)$ *satisfying* $p^{1e} \geq p^{2e} + \epsilon$ *and* $(p^{1e}, p^{2e}) = ((6b + 10c + 3\epsilon)/16, (8b + 8c + 4\epsilon)/16)$ *satisfying* $p^{1e} \leq p^{2e} - \epsilon$.

The symmetric equilibrium points to an equal market split. In non-symmetric equilibria, the retailer charging more has a smaller market share. Although both retailers are identical, the market can have a retailer leading with a higher price and the other leading with a larger market share.

When the WTPs are dependent as opposed to independent, we obtain lower equilibrium prices. Using $\mathrm{P}(W^i \geq p^i) = (\epsilon + 2(b - p^i))/(2b)$ and loose coupling, the equilibrium price is $p^{ie} = (2b + 2c + \epsilon)/4$ for uniformly distributed independent WTPs and identical retailers. When the WTPs are dependent, the symmetric equilibrium price $p^{1e} = p^{2e} = (6b + 9c + 4\epsilon)/15$ is lower than $(2b + 2c + \varepsilon)/4$. Similarly the non-symmetric equilibria satisfy $(p^{1e} + p^{2e})/2 < (2b + 2c + \epsilon)/4$. Recognition of dependence in our example reduces prices, which is a welcome news to customers but not so to firms. Ideally, firms should reduce the dependence of WTPs, possibly by employing product differentiation strategies.

3.3. Dependent and Discrete-Valued Willingness-to-Pays Lead to Price Cycles

A price cycle is a dynamic price equilibrium identified by a finite sequence of non-identical multiple price-pairs, which satisfies three conditions: (i) any consecutive pair must share a *common price*; (ii) the *uncommon price* in the succeeding pair is the best response to the *common price*; (iii) when the last and the first price pairs in the sequence are considered as consecutive price pairs, their prices satisfy conditions (i) and (ii). We consider discrete-valued finite WTPs in this section because they can cause a price cycle. Discrete WTPs imply discrete prices—often found in practice as multiples of ¢1 or ¢5 (Phillips 2005).

In this study, we seek only pure strategy equilibria and always refer to *pure strategy equilibrium* when saying *equilibrium*. In the absence of a price-pair equilibrium, it is possible to seek a price cycle that consists of multiple price-pairs. One may also seek a mixed strategy equilibrium in the absence of a pure strategy equilibrium, however, we choose to focus only on pure strategies. When there is not a price-pair equilibrium, we can start at an arbitrary price-pair and generate a price sequence that satisfies (i) and (ii). Since price-pairs are finite in our discrete case, such a sequence must also satisfy (iii). Hence, absence of a price-pair equilibrium implies the presence of at least one price cycle. The contrapositive of this statement is also true; absence of a price cycle implies the presence of a price-pair equilibrium. Moreover, price cycles and price-pair equilibria may coexist in a given instance. Consequently, a price cycle can be the best description of the equilibrium in a market that does not have a price-pair equilibrium.

A price cycle can be represented by a sequence of price-pairs: $\{p_0^1, p_0^2\} \rightarrow \{p_0^1, p_1^2\} \rightarrow \{p_1^1, p_1^2\} \rightarrow \{p_1^1, p_2^2\} \rightarrow \cdots \rightarrow \{p_j^1, p_0^2\} \rightarrow \{p_0^1, p_0^2\}$, where "→" indicates the direction of a price cycle. For example, $\{p_1^1, p_1^2\} \rightarrow \{p_1^1, p_2^2\}$ implies that the cycle goes from $\{p_1^1, p_1^2\}$ to $\{p_1^1, p_2^2\}$, as p_2^2 is the best response of the retailer 2 to retailer 1's price p_1^1. The length of the price cycle is the minimum number of price-pairs traversed before returning to the same price-pair. Accordingly, the shortest price cycle is of length 4.

An example of a price cycle with length 4 is depicted by arrows in Table 2, which shows the joint WTP probabilities for prices $p^1 \in \{1, 2, 7\}$ and $p^2 \in \{1, 2, 3\}$, respectively, charged by retailers 1 and 2. In the example, retailers incur zero cost and $\phi = 0.4$. The expected profits of the retailers are from Equation (8), e.g., $\Pi_1(2, 2) = 2(\phi \mathrm{P}(W^1 \geq 2) + (1 - \phi)\mathrm{P}(W^1 \geq 2, W^2 < 2)) = 2(0.4(0.7) + 0.6(0.3)) = 0.92$, other profits are in Table 2. They satisfy $\Pi_2(2, 3) > \Pi_2(2, 1), \Pi_2(2, 2)$; $\Pi_1(7, 3) > \Pi_1(1, 3), \Pi_1(2, 3)$; $\Pi_2(7, 2) > \Pi_2(7, 1), \Pi_2(7, 3)$ and $\Pi_1(2, 2) > \Pi_1(1, 2), \Pi_1(7, 2)$, and these four inequalities respectively justify the four arrows in the cycle $\{2, 2\} \rightarrow \{2, 3\} \rightarrow \{7, 3\} \rightarrow \{7, 2\} \rightarrow \{2, 2\}$. This is the unique cycle and there does not exist a single price-pair equilibrium.

The dependence of WTPs can eliminate a price-pair equilibrium and leads to a price cycle as in the above example. If they are independent, there is always a price-pair equilibrium by Theorem 1. On the other

Table 2 Price Cycle Example. Willingness To Pay probabilities $\mathrm{P}(\mathbf{W}^1 = \mathbf{p}^1, \mathbf{W}^2 = \mathbf{p}^2)$; **Profits** $(\Pi_1(\mathbf{p}^1, \mathbf{p}^2), \Pi_2(\mathbf{p}^1, \mathbf{p}^2))$

		Retailer 2		
	(p^1, p^2)	1	2	3
Retailer 1	1	0.00; (0.40, 0.60)	0.05; (0.58, 0.84)	0.25; (0.73, 0.81)
	2	0.25; (0.56, 0.72)	0.10; (0.92, 1.08) →	0.10; (1.16, 1.11) ↓
	7	0.05; (0.70, 0.90)	0.10; (0.91, 1.24) ↑	0.10; (1.33, 1.23) ←

hand, even if there is a price-pair equilibrium, WTPs can be dependent. This is because WTP dependence can be induced by altering WTP probabilities that do not show up in the equilibrium comparisons. So, inferring independence is harder, but still possible as in parts (a) and (b) of the next theorem. It is also important to characterize the absence of a price cycle toward concluding that a price-pair equilibrium exists under dependent WTPs, this reasoning is adopted by the theorem.

THEOREM 2 (EQUILIBRIUM WITH DEPENDENT WTPS).

(a) *There is no price cycle of length 4 such as* $\{p_l^1, p_l^2\} \to \{p_l^1, p_h^2\} \to \{p_h^1, p_h^2\} \to \{p_h^1, p_l^2\} \to \{p_l^1, p_l^2\}$, *if the WTPs satisfy*

$$\mathrm{P}(W^i \geq p_l^i, W^{-i} < p_h^{-i})\mathrm{P}(W^i \geq p_h^i, W^{-i} < p_l^{-i}) = \mathrm{P}(W^i \geq p_h^i, W^{-i} < p_h^{-i})\mathrm{P}(W^i \geq p_l^i, W^{-i} < p_l^{-i}).$$

(b) *The WTPs are independent if the condition in (a) is satisfied for all prices.*

(c) *No price cycle of length 4 can contain a price pair with the lowest prices for both retailers. Consequently, there is a price-pair equilibrium if both retailers consider binary prices* $p^i \in \{p_l^i, p_h^i\}$.

(d) *There is no price cycle of length 4 if WTPs and preferences satisfy*

$$\max_{\{p_l^i, p_h^i, p^{-i} \in \{p_l^{-i}, p_h^{-i}\}\}} \left\{ \frac{\mathrm{P}(p_h^i \leq W^i, p^{-i} > W^{-i}) + \phi_i \mathrm{P}(p_h^i \leq W^i, p^{-i} \leq W^{-i})}{\mathrm{P}(p_l^i \leq W^i < p_h^i, p^{-i} > W^{-i}) + \phi_i \mathrm{P}(p_l^i \leq W^i < p_h^i, p^{-i} \leq W^{-i})} \right\} \leq \frac{a_i - c_i}{b_i - a_i}$$

for either $i = 1$ *or* 2.

(e) *There is a price-pair equilibrium if either one of the retailers considers binary prices* $p^i \in \{p_l^i, p_h^i\}$ *and the condition in (d) is satisfied.*

Theorem 2(a) gives a condition to eliminate a particular cycle. This condition boils down to independence of WTPs when all cycles of length 4 are to be ruled out. So, independence is sufficient to eliminate these cycles. Theorem 2(a)–(b) eliminate the cycles whereas Theorem 1 establishes the existence of a price-pair equilibrium, which does not rule out cycles. Theorem 2(c) shows that when retailers consider binary prices, there must be a price-pair equilibrium despite the dependence of the WTPs. Theorem 2(d) gives the condition under which there is no price cycle of length 4 despite the dependence of WTPs. From the WTP distribution in Table 2, we evaluate the left-hand side of the condition in Theorem 2(d) for $i = 1$ and $i = 2$. Correspondingly, if either $\frac{39}{15} \leq \frac{a_1}{b_1 - a_1}$ or $\frac{37}{17} \leq \frac{a_2}{b_2 - a_2}$ holds, there are no price cycles of length 4. These inequalities imply conditions on the support parameters $\frac{a_1}{b_1} \geq \frac{39}{54}$ or $\frac{a_2}{b_2} \geq \frac{37}{54}$. So, if the support of either W^1 or W^2 is tight, i.e., the uncertainty of W^1 or W^2 is low, there are no price cycles of length 4. This conclusion leads to price-pair equilibrium in Theorem 2(e) when either retailer considers binary prices. In these regards, Theorem 2 applies even when Theorem 1 does not.

3.4. Price-Dependent Preference

Another extension of WTP-choice model involves price-dependent preference $\phi(p^1, p^2)$ that can be used to capture some of the real-life contexts, where not only profits but also price responses are coupled. In a duopoly profit maximization problem with continuous WTPs, retailer 1's objective is $\max_p (p - c_1)(1 - W_1(p))\{\phi(p, p^2) + (1 - \phi(p, p^2))W_2(p^2)\}$. The best response price p^1 for retailer 1 satisfies:

$$1 = \frac{c_1}{p^1} + \left[\Lambda_1(p^1) - \frac{p^1(1 - W_2(p^2))[\partial\phi(p^1, p^2)/\partial p^1]}{\{\phi(p^1, p^2) + (1 - \phi(p^1, p^2))W_2(p^2)\}}\right]^{-1}. \quad (9)$$

Table 3 Examples with Dependent Preferences

Preference	Response price
Price-independent preference ϕ	p^1 is given by $1 = c_1/p_1 + [\Lambda_1(p^1)]^{-1}$
Increasing in p^1 $$\phi(p^1, p^2) = \frac{p^1}{p^1 + p^2}$$	p^1 is higher than the price under price-independent preference $$1 = \frac{c_1}{p^1} + \left[\Lambda_1(p^1) - \frac{p^1 p^2(1 - W_2(p^2))}{(p^1 + p^2)\{p^1 + p^2 W_2(p^2)\}}\right]^{-1}$$
Decreasing in p^1 with parameters p^1_{max}, p^2_{max} $$\phi(p^1, p^2) = \frac{p^1_{max} - p^1}{p^1_{max} + p^2_{max} - p^1 - p^2}$$	p^1 is lower than the price under price-independent preference $$1 = \frac{c_1}{p^1} + \left[\Lambda_1(p^1) + \frac{p^1 p^2(1 - W_2(p^2))}{\{p^1_{max} p^2_{max} W_2(p^2) - p^2(p^1_{max} - p^1)(1 - W_2(p^2))\}}\right]^{-1}$$

It is easy to see from above that the best response p^1 for retailer 1 depends on other retailer's price p^2, unlike loose coupling in Theorem 1. This coupling can lead to higher or lower price responses and is shown with two examples in Table 3. In particular, a retailer responds by charging a higher (lower) price if the preference for that retailer is increasing (decreasing) in its own price compared to the price charged when the preference is independent of prices.

THEOREM 3 (EQUILIBRIUM WITH PRICE-DEPENDENT PREFERENCES). *If the preferences for both retailers are increasing (decreasing) in their prices and an equilibrium exists, then the equilibrium prices will be higher (lower) compared to the equilibrium prices when the preferences are independent of prices.*

4. Centralized Pricing with WTP-Choice Model

In the centralized pricing context, a retailer sells both products 1 and 2, and maximizes the profit

$$\begin{aligned}\Pi(p^1,p^2) &= \bar{W}_1(p^1)\{(p^1-c_1)(1-\phi_2\bar{W}_2(p^2)) \\ &\quad -\phi_1(p^2-c_2)\bar{W}_2(p^2)\}+(p^2-c_2)\bar{W}_2(p^2) \\ &= \bar{W}_2(p^2)\{(p^2-c_2)(1-\phi_1\bar{W}_1(p^1)) \\ &\quad -\phi_2(p^1-c_1)\bar{W}_1(p^1)\}+(p^1-c_1)\bar{W}_1(p^1),\end{aligned}$$

where we assume the independence of WTPs and use the tail probability $\bar{W}_i$ for W^i of product i. In the remainder of the paper, we consider independent WTPs because the solution with independent WTPs can approximate the solution with dependent WTPs and the WTP-choice model with independent WTPs represent real-life data sufficiently well as discussed in section 6.

Since $\Pi(p^1, p^2)$ is continuous in both of its arguments, it has one or possibly many global maximizers. Interestingly, the centralized pricing problem turns out to be more challenging than the (decentralized) pricing game, whose analysis simplifies due to loose coupling. Recall that the support of WTP for product i is $[a_i, b_i]$, so the profit $\Pi(p^1, p^2)$ in general can be defined *inside* as well as *outside* the WTP region $\{(p^1, p^2) : p^1 \in [a_1, b_1] \text{ and } p^2 \in [a_2, b_2]\}$. Similar to previous sections, we let $\mathcal{P}_i(p) = \arg\max_{p^i} \Pi(p^i, p^{-i} = p)$ for $0 \le p < \infty$. This maximizer is unique, in the interval $[a_i, b_i]$ and depends on preference ϕ_i as shown in the next theorem, which is the counterpart of Theorem 1. Therefore, the centralized pricing solution is obtained from the intersection of $\mathcal{P}_1(p^2)$ and $\mathcal{P}_2(p^1)$. Consequently, a unique intersection of $\mathcal{P}_1(p^2)$ and $\mathcal{P}_2(p^1)$ leads to the unique optimal price pair.

THEOREM 4 (CENTRALIZED PROFIT AND PRICES).

(a) *The profit Π is unimodal in p^i when p^{-i} is fixed. Its maximizer $\mathcal{P}_i(p^{-i})$ is unique and in the interval $[a_i, b_i]$. When $a_i < p = \mathcal{P}_i(p^{-i}) < b_i$, it solves*

$$\frac{w_i(p)}{\bar{W}_i(p)}\left\{p-c_i-(p^{-i}-c_{-i})\frac{\phi_i}{1/\bar{W}_{-i}(p^{-i})-(1-\phi_i)}\right\}=1. \tag{10}$$

$\mathcal{P}_1(p^2)$ and $\mathcal{P}_2(p^1)$ must intersect at least once inside the WTP region.

(b) *Centralized prices are higher than their decentralized (equilibrium) counterparts. A preferred product is priced higher: $\mathcal{P}_i(p^{-i})$ increases in ϕ_i.*

We propose the Centralized Iterative Pricing Algorithm (CIPA) in Table 4 to find the intersection of $\mathcal{P}_1(p^2)$ and $\mathcal{P}_2(p^1)$. Afterwards, we prove that CIPA gives the global optimal for the centralized profit under some assumptions that will be addressed later on.

THEOREM 5 (GLOBAL OPTIMAL FROM CIPA). *If $\mathcal{P}_1(p^2)$ and $\mathcal{P}_2(p^1)$ intersect exactly once and the CIPA converges, then CIPA yields the global optimal for the centralized profit.*

Centralized Iterative Pricing Algorithm can numerically be checked in each instance to see if it delivers a converging sequence $\{(p^1_n, p^2_n)\}$. Similarly, it may be possible to draw $\mathcal{P}_1$ and $\mathcal{P}_2$ in an instance to find intersection point(s). Rather than resorting to numerical analysis, we push the arguments analytically by considering two special cases of the WTP distributions: uniform and shifted exponential. The next theorem shows in these cases that $\mathcal{P}_1$ and $\mathcal{P}_2$ intersect exactly once and satisfy the hypothesis of Theorem 5.

THEOREM 6 (UNIQUE INTERSECTION OF $\mathcal{P}_1$ AND $\mathcal{P}_2$).

(a) *If the WTP of product i has a uniform distribution over $[a_i, b_i]$ for $i \in \{1, 2\}$, then*

Table 4 Centralized Iterative Pricing Algorithm

Inputs: Functions $\mathcal{P}_1$, $\mathcal{P}_2$ and accuracy parameter $\varepsilon > 0$.
Initialize $p^1_1 = a_1$, $p^2_1 = \mathcal{P}_2(p^1_1)$, $n = 1$.
Repeat $n = n + 1$;
$p^1_n = \mathcal{P}_1(p^2_{n-1})$ and $p^2_n = \mathcal{P}_2(p^1_n)$
Until prices satisfy $|p^2_n - p^2_{n-1}| \le \epsilon$.
Outputs: Sequence of $\{(p^1_n, p^2_n)\}$.

$$\mathcal{P}_i(p^{-i}) = \frac{c_i}{2} + \frac{b_i}{2} + \frac{\phi_i}{2}(p^{-i} - c_{-i}) \frac{1}{(b_{-i} - a_{-i})/(b_{-i} - p^{-i}) - \phi_{-i}}, \tag{11}$$

which decreases in p^{-i}. In addition, if $b_i/a_i \geq (1 - \phi_i c_i/a_i)/(1 - \phi_i)$, then $p^i - \mathcal{P}_i(\mathcal{P}_{-i}(p^i))$ is increasing. It crosses zero exactly once at $p^i \geq \max\{a_i, (b_i + c_i)/2\}$, i.e., $\mathcal{P}_1$ and $\mathcal{P}_2$ intersect exactly once.

b) If the WTP of product i has a shifted exponential distribution $(a_i, \infty; \tau_i)$ for $i \in \{1, 2\}$, then

$$\mathcal{P}_i(p^{-i}) = c_i + \frac{1}{\tau_i} + \phi_i(p^{-i} - c_{-i}) \frac{1}{e^{\tau_{-i}(p^{-i} - a_{-i})} - \phi_{-i}}, \tag{12}$$

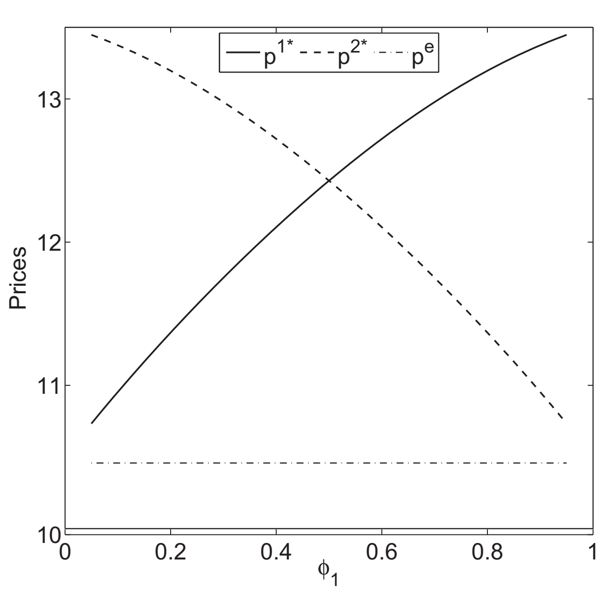
which decreases in p^{-i}. In addition, if $(a_i - c_i)\tau_i + \sinh^{-1}(\phi_i) \leq 1$, then $p^i = \mathcal{P}_i(\mathcal{P}_{-i}(p^i))$ has a unique fixed point $p^i \geq \max\{a_i, 1/\tau_i + c_i\}$, i.e., $\mathcal{P}_1$ and $\mathcal{P}_2$ intersect exactly once.

Theorem 6 has two technical conditions $b_i/a_i \geq (1 - \phi_i c_i/a_i)/(1 - \phi_i)$ and $(a_i - c_i)\tau_i + \sinh^{-1}(\phi_i) \leq 1$. Both are satisfied when $a_i = c_i$. For example, $(a_i - c_i)\tau_i + \sinh^{-1}(\phi_i) \leq 1$ holds for $a_i = c_i$ as $\phi_i \leq 1 < 1.175 \approx \sinh(1)$. The theorem establishes the uniqueness of intersection of $\mathcal{P}_1(p^2)$ and $\mathcal{P}_2(p^1)$ for two important WTP distributions by using two related proof methodologies of monotonicity for $p - \mathcal{P}_i(\mathcal{P}_{-i}(p))$ and fixed point for $p - \mathcal{P}_i(\mathcal{P}_{-i}(p)) = 0$. It also gives us the price $\mathcal{P}_i(p^{-i})$ expressions from which we can observe that this price increases in ϕ_i, b_i, $1/\tau_i$, a_{-i}, b_{-i}, $1/\tau_{-i}$ and decreases in p^{-i}, ϕ_{-i}. Most of these are intuitive as they imply that the price of a product increases with its WTP and preference when the price of the other product is constant. When both prices are optimized, some interesting results are illustrated below through numerical examples.

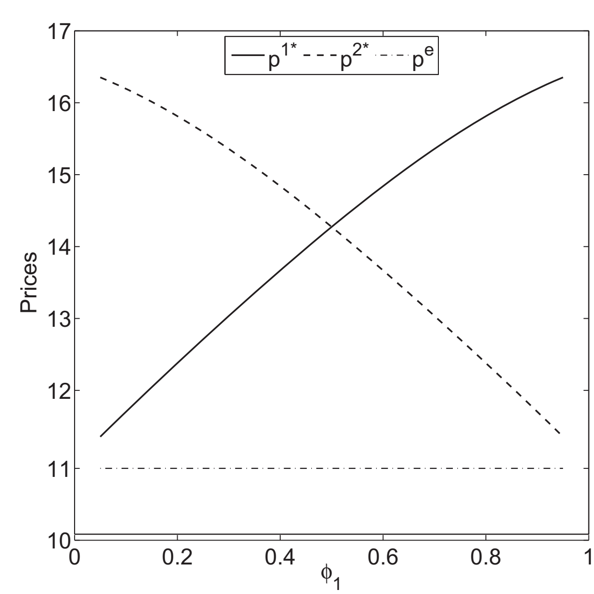
Centrally optimal prices (p^{1*}, p^{2*}) and equilibrium prices (p^{1e}, p^{2e}) can be computed and compared under various market parameters: (a_i, b_i, ϕ_i) for uniform distribution and (a_i, τ_i, ϕ_i) for the shifted exponential distribution. In our base case, the parameter values are $(a_i, b_i, \phi_i) = (5, 20, 0.5)$ and $(a_i, \tau_i, \phi_i) = (5, 0.1, 0.5)$ and $c_i = 1$ for $i = 1, 2$ and these values are altered one by one as indicated by the horizontal axes of Figures 4a–6b. In these figures, as in Theorem 4 (b), competition among retailers drives the prices down. The centrally optimal price p^{1*} increases and p^{2*} decreases when the preference ϕ_1 increases (Figure 4a and b). The equilibrium prices of Table 1 are illustrated in Figures 4a,b and 5a,b and do not depend on parameters ϕ_1 or a_1.

In Figure 5a and b, the optimal price p^{1*} decreases with higher a_1. This is counter to the intuition as higher a_1 implies a (stochastically) larger WTP for product 1 and in turn should lead to a non-decreasing price for product 1 in the single-product context. In the two-product context, however, p^{2*} increases with a_1 and $p^{1*} = \mathcal{P}_1(p^{2*})$ decreases with p^{2*}. The cumulative effect turns out to be that p^{1*} decreases with a_1. Hence, Figure 5a and b are important in illustrating the interaction between two products that cannot be inferred from the single-product context.

Figure 4 Optimal Prices (p^{1*}, p^{2*}) and Equilibrium Prices ($p^{1e} = p^{2e} = p^e$) as Preference ϕ_1 Increases

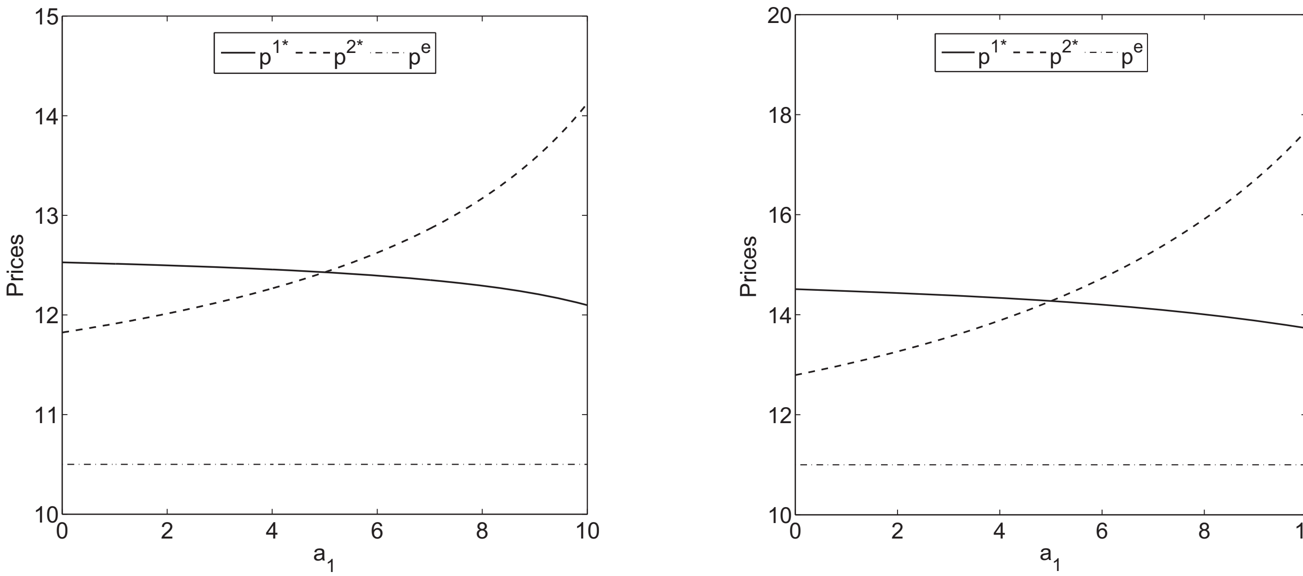


(a) Uniform WTP



(b) Shifted-exponential WTP

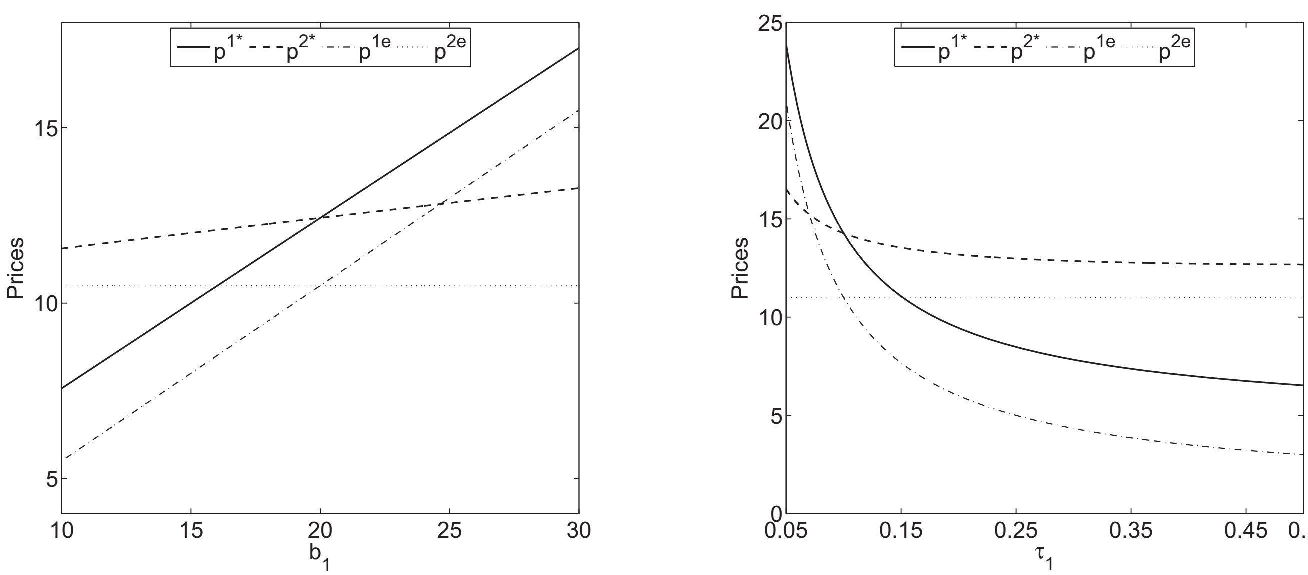
Figure 5 Optimal Prices (p^{1*}, p^{2*}) and Equilibrium Prices ($p^{1e} = p^{2e} = p^e$) as the Least WTP a_1 Increases



(a) Uniform WTP

(b) Shifted-exponential WTP

Figure 6 Optimal Prices (p^{1*}, p^{2*}) and Equilibrium Prices (p^{1e}, p^{2e}) as the Parameter b_1 or τ_1 Increases



(a) Uniform WTP with b_1

(b) Shifted-exponential WTP with τ_1

Similar to a_1, an increase in b_1 or in $1/\tau_1$ increases the WTP for product 1. In view of Equations (11–12), both b_1 and $1/\tau_1$ have a direct effect on $\mathcal{P}_1(p^2)$ and increase it, unlike a_1. Indirectly, higher b_1 pulls $\mathcal{P}_2(p^1)$ up and in turn pushes $\mathcal{P}_1(p^2)$ down. In Figure 6a and b, the direct effect dominates the indirect effect and the optimal prices (p^{1*}, p^{2*}) increase in b_1 and $1/\tau_1$. The equilibrium price p^{1e} increase with b_1 and $1/\tau_1$ in accordance with Table 1 while the equilibrium price p^{2e} is constant in b_1 and $1/\tau_1$ as implied by the loose coupling property.

It is easy to check $\Pi(p^1, p^2) = \Pi_1(p^1, p^2) + \Pi_2(p^1, p^2)$ for every p^1, p^2. Hence, $\Pi_1(p^{1*}, p^{2*}) + \Pi_2(p^{1*}, p^{2*}) - \Pi_1(p^{1e}, p^{2e}) + \Pi_2(p^{1e}, p^{2e}) = \Pi(p^{1*}, p^{2*}) - \Pi(p^{1e}, p^{2e}) \geq 0$, i.e., the total profit is higher in the centralized pricing context than that in the competitive context.

5. Competitive Pricing under Stockouts

Consideration of stockouts (inventory unavailability) can improve the applicability of a choice model. A stockout at a retailer increases the demand at another retailer as customers substitute their preferred but stocked-out product with another. We can investigate the versatility of WTP-choice model and test the robustness of loose coupling under stockouts and competition. We continue to consider independent WTPs because the loose coupling property fails already with dependent WTPs.

We use fill rates to study competitive pricing under stockouts in a duopoly. If a customer arrives at a stocked-out retailer, he naturally considers buying from another retailer; this can be called inventory-based substitution behavior. Otherwise, if stockout-facing customers do not consider buying from another retailer and simply buy nothing, the expected demand faced by a retailer is simply his fill rate times the demand without stockouts. Subsequently, competitive pricing follows the same structure (including loose coupling) as before and is not interesting to analyze.

Under inventory-based substitution, customers preferring retailer 2 in particular, consider retailer 1 when retailer 2 is stocked-out. Previously, such customers considered retailer 1 only when retailer 2 prices too high. Now both a stockout and a high price at retailer 2 divert customers to retailer 1, hence the choice probability of retailer 1 needs to have some additional terms based on stockouts. These additional terms are detailed below for a lost sales case. The stockout probability or *stockout rate* at retailer i is denoted by v_i , i.e., $1 - v_i$ is the fill rate of retailer i.

A customer sooner or later, depending on preferring retailer i or the other, shows up at retailer i with the probability $\phi_i + \phi_{-i}[(1 - v_{-i})W_{-i}(p^{-i}) + v_{-i}]$. Note that these probabilities arise only under the lost sales assumption, where a customer facing stockout does not wait or backorder with the retailers. This customer finds the product in stock with fill rate $1 - v_i$ and buys with probability $1 - W_i(p^i)$. Hence, the sales probability is

$$\rho_i^{ls}(p^1, p^2) := (1 - v_i)(1 - W_i(p^i)) \times [\phi_i + \phi_{-i}((1 - v_{-i})W_{-i}(p^{-i}) + v_{-i})].$$

When stockouts are considered, we use the term sales probability rather than choice probability. The difference between a choice and a sale is the inventory availability incorporated via v_i. If both retailers are stocked-out or price too high for a particular customer, they lose the customer. This happens with probability $[v_i + (1 - v_i)W_i(p^i)][v_{-i} + (1 - v_{-i})W_{-i}(p^{-i})]$. Since $v_i + (1 - v_i)W_i(p^i) \geq W_i(p^i)$, the loss probability is at least $\rho^0(p^1, p^2) = W_1(p^1)W_2(p^2)$ of section 2 for $\delta = 1$. Eventually, the profit of retailer i under lost sales is

$$\Pi_i^{ls}(p^1, p^2) := (p^i - c_i)\rho_i^{ls}(p^1, p^2).$$

Equipped with profit expressions, we can express the response price for retailer i:

$$\mathcal{P}_i^{ls}(p^{-i}) = \arg\max_{p^i} \Big\{ \Pi_i^{ls}(p^1, p^2) = (p^i - c_i)(1 - W_i(p^i)) (1 - v_i)[\phi_i + \phi_{-i}((1 - v_{-i})W_{-i}(p^{-i}) + v_{-i})] \Big\}.$$

The profit Π_i^{ls} has multiplication of three terms, the first two are $(p^i - c_i)$ and $1 - W_i(p^i)$, and they depend on retailer i's price p^i while the last term in brackets is not dependent on p^i. So, this profit expression has the same structure as Equation (5). Hence, the price solution $p_{ls}^{i,e}$ is the same as that without stockouts as in Equation (6): $p_{ls}^{i,e} = c_i + \bar{W}_i(p_{ls}^{i,e})/w_i(p_{ls}^{i,e})$. The prices of Table 1 as well as loose coupling still remain valid. More interestingly, the equilibrium price charged by a retailer does not depend on the stockout rates, as the price is relevant only when the retailer can fulfill the demand. However, the profit Π_i^{ls} depends heavily on both v_i and v_{-i}.

As shown above, consideration of inventory unavailability under the lost sales assumption does not harm the loose coupling property and Theorem 1 still applies. Emboldened by this observation, Gupta (2014) presents two backorder models obtained by considering whether a stock-out facing customer at his preferred retailer gives priority to his preference and backorders from the preferred retailer or to immediate availability and attempts to buy a similar product from another retailer. Interestingly, loose coupling property continues to hold in both of the backorder models. In sum, the loose coupling property is robust with respect to the consideration of inventory unavailability through various models.

6. Estimation with Scanner Data and Numerical Comparisons

We use the maximum (log-) likelihood estimation (MLE) scheme with real-life data pertaining to low-price items, whose choices are likely to be based more on preferences than on utility maximization. We report the mean percentage errors (MPEs) in sales to compare various parameterizations of WTP-choice model with standard and mixed Logit models. Given the scanner data of prices $\{p_n = [p_n^1, \ldots, p_n^M] : n = 1, \ldots, N\}$ and of choices $\{y_n = [y_n^1, \ldots, y_n^M] :$

$n = 1, \ldots, N\}$ along with preferences $\Phi = [\phi_1, \ldots, \phi_S]$ over consideration sets $[\mathfrak{L}_1, \ldots, \mathfrak{L}_S]$, WTPs $W = [W_1, \ldots, W_M]$, $\alpha = [\alpha^1, \ldots, \alpha^M]$ and β, the log-likelihood functions for WTP-choice, standard logit and mixed logit models are

$$L_{wtp}(\delta, \Phi, W) = \sum_{n=1}^{N} \sum_{m=0}^{M} y_n^m \log \rho_n^m(\delta, \Phi, W; p_n, y_n),$$

$$L_{sl}(\alpha, \beta) = \sum_{n=1}^{N} \sum_{m=0}^{M} y_n^m \log \lambda_n^m(\beta, \alpha; p_n, y_n),$$

$$L_{ml}(\alpha, F) = \sum_{n=1}^{N} \sum_{m=0}^{M} y_n^m \log \frac{1}{B} \sum_{b=1}^{B} \lambda_n^m(\beta_b, \alpha; p_n, y_n),$$

where $y_n^0 = 1 - \sum_m y_n^m$, ρ_n^m is from Equations (2–3) or (4) and λ_n^m is from (1), and α_0 is set to zero in Logit model without any loss of generality. For WTP-choice model, the likelihood formulation above uses independent WTPs. This formulation can be extended for dependent WTPs and the resulting L_{wtp} can be maximized, but such a maximization would require more computational effort and can be avoided unless necessary. For mixed Logit model, we follow the simulated log-likelihood function obtained numerically through simulation (Revelt and Train 1998), where B is the number of draws from the cumulative density function $F(\beta)$ giving $\beta_1, \ldots, \beta_B$.

In Lemma 3 of Appendix S1, we establish the concavity of $L_{wtp}(\delta, \Phi, W)$ for given W and concavity of the L_{wtp} for $\llcorner$ distribution in each of the WTP parameters τ_m or a_m. Without a readily available package to maximize L_{wtp}, we develop a simple search technique for maximization by assuming that either parameters are discrete or are approximated well by discrete values. We identify sets of values for parameters (e.g., δ, ϕ, a_1, a_2, b_1, b_2 for $\square$ WTP-choice model with $M = 2$), whose Cartesian product yield the parameter space. With *R* (www.r-project.org), we compute the L_{wtp} functions over the parameter space to find the maximizer.

6.1. Real-Life Data

We have obtained scanner data for *candy melts* from retailer $\mathcal{X}$, which has limited the data exposure. Retailer $\mathcal{X}$ sells various types of candy melts. For estimation with $M = 2$, we consider dark and light (regular) chocolate candy melts. The sales data are weekly, cover about a year and a half, and consist of 14,940 purchases. Retailer $\mathcal{X}$ usually keeps the candy melt prices constant over a week, so the total revenue divided by the total sales for each product in each week is a good indicator of that week's price. The sales and prices for chocolate candy melts are in Figure 7, where no-purchases are customers that did not purchase one of the chocolate candy melts under consideration. For $M = 3$, mint chocolate candy melt is considered in addition to dark and light chocolate candy melts. We also use three publicly available data sets on "yogurt," "ketchup," and "tuna." These items have low prices and high purchase frequencies—two factors that may favor WTP-choice model.

Yogurt data (Jain et al. 1994) consists of 2006 observations. For $M = 2$, we focus on two common brand choices of *Dannon* (818 purchases) and *Yoplait* (674 purchases). The remaining 514 customers are put under no-purchase.

Dannon and *Yoplait* prices, respectively, range over \$1.9–\$11.1 and \$0.3–\$19.3. For $M = 3$, *Hiland* (44 purchases) is the third product with prices over \$2.5–\$7.6.

Ketchup data (Kim et al. 1995) consists of 4956 observations. For $M = 2$, we focus on *Heinz* (2526 purchases) and *Del Monte* (256 purchases) whose prices respectively range over \$0.79–\$1.47 and \$0.89–\$1.49. For $M = 3$, the *store brand* (1155 purchases) is the third product with prices over \$0.75–\$0.99.

Canned *Tuna data* (Kim et al. 1995) consists of 13,705 observations. For $M = 2$, we focus on *Sko* (2439 purchases) and *Cosw* (2238 purchases) whose per-pound prices, respectively, range over \$0.29–\$0.89 and \$0.19–\$0.99. For $M = 3$, we also consider *Pw* (1050 purchases) with prices over \$0.17–\$0.70.

The likelihood function $L_{wtp}(\delta, \phi, a_1, b_1, a_2, b_2)$ for $\square$ WTPs has six parameters. To reduce the dimension of the maximization problem, we can separate the estimation of a_1 and a_2 from the rest. For $\square$ WTPs, the minimum price p_n^m (at which m is sold) is the MLE of a_m, i.e., $a_m^0 := \min\{p_n^m : y_n^m = 1\}$, but then the L_{wtp} has terms such as $\log(p_n^m - a_m^0)$ that become negative

Figure 7 **Candy Melt Data for $M = 2$. Left: Weekly sales (–) and prices (- -) for dark chocolate; Middle: Weekly sales (–) and prices (- -) for light chocolate; Right: Weekly no-purchases (–)**

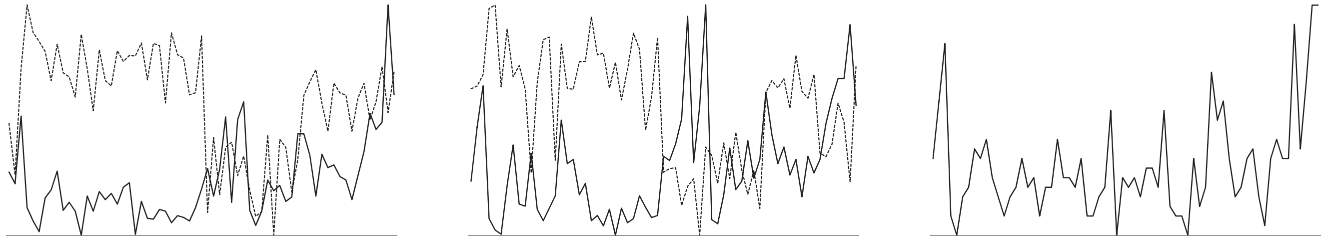


Table 5 Log-Likelihoods for Standard and Mixed Logit Models and (△, $\delta \le 1$) WTP-Choice Model

	Standard logit	Mixed logit	(△, $\delta \le 1$) WTP-choice
Yogurt $N = 1757$	$L_{sl} = -1835$	$L_{ml} = -1832$	$L_{wtp} = -1836$
Ketchup $N = 4564$	$L_{sl} = -4169$	$L_{ml} = -4162$	$L_{wtp} = -4152$
Candy melt $N = 14{,}125$	$L_{sl} = -11{,}498$	$L_{ml} = -11{,}496$	$L_{wtp} = -11{,}560$
Tuna $N = 13{,}332$	$L_{sl} = -11{,}143$	$L_{ml} = -11{,}143$	$L_{wtp} = -11{,}024$

infinity when $p_n^m = a_m^0$ for customer i. To avoid this numerical difficulty, we remove those customers who bought product m at the minimum price of a_m^0 and end up with the N (number of purchases) values in Table 5.

6.2. Likelihood Comparisons of the Choice Models

We consider six different WTP-choice models by setting $\delta = 1$ or $\delta \le 1$ and parameterize WTPs with a uniform (□), shifted exponential (∟) or triangle (△) distribution. For example, △ WTP and $\delta \le 1$ make up a special WTP-choice model denoted by (△, $\delta \le 1$), and similar notation applies to the others. These distributions have positive supports; □ and ∟ distributions have increasing generalized failure rates; □ distribution can represent a firm uninformed about its customers; ∟ and △ distributions can represent an informed firm that matches the mode of a product's WTP distribution with the reference price of that product. This parameterization flexibility of WTP-choice model is its advantage over Logit model, which allows only double-exponentially distributed utilities. We also set $M = 2$ until prediction accuracy tests.

First, we compare standard Logit model, mixed Logit model, and (△, $\delta \le 1$) WTP-choice model. The mixed Logit model has the parameters $(\alpha_1, \alpha_2, \mu_\beta, \sigma_\beta)$, where μ_β and σ_β are, respectively, the mean and standard deviation of the uniformly distributed price coefficient β. △ WTP-choice model has parameters $(\delta, \phi, a_1, b_1, m_1, a_2, b_2, m_2)$ but we reduce the number of estimated parameters by fixing $a_i = a_i^0$. Table 5 gives the estimates for parameters and the corresponding L values.

From Table 5, △ WTP-choice model gives a better (higher) L value for ketchup and tuna data compared to mixed Logit model. L values of -1832 and -1836 for yogurt data are very close to each other and the same is true (to a lesser extent) for the candy melt data. L value of the mixed Logit model is smaller than that of WTP-choice model for two data sets. When L value is larger, it is so at most by 0.55%. So, (△, $\delta \le 1$) WTP-choice model performs as well as the mixed Logit model.

Since WTP-choice model in Table 5 has a △ WTP distribution, we need to estimate the mode of this distribution. Instead, if we use a □ WTP distribution, the mode is not required as the support suffices. This reduces the number of parameters by one for each product. Another reduction is obtained by assuming $\delta = 1$, i.e., every customer is interested in buying either one of the products and checks the prices. While reducing the parameters down to (ϕ, b_1, b_2), (□, $\delta = 1$) WTP-choice model also reduces the L_{wtp}. Such decreases are reported as Change in % in Table 6, where Change in % $:= [L_{wtp}(\triangle, \delta \le 1) - L_{wtp}(\square, \delta = 1\text{or}\ , \delta = 1)]/L_{wtp}(\triangle, \delta \le 1)$ and $L_{wtp}(\triangle, \delta \le 1)$ values are from Table 5. Similarly, we report Difference in % $:= [L_{ml} - L_{wtp}(\square, \delta = 1\text{or}\ , \delta = 1)]/L_{ml}$. Hence, negative values of Change in % and Difference in % indicate a drop from the L values in Table 5.

According to Table 6, (□, $\delta = 1$) and (∟, $\delta = 1$) WTP-choice models are outperformed by mixed Logit model. On the other hand, the performance of (□, $\delta = 1$) WTP-choice model relative to (△, $\delta \le 1$) WTP-choice model suffers significantly with ketchup and tuna data. In other words, WTPs of ketchup and tuna customers resemble △ distribution better than □ distribution, and some of these customers are not interested ($\delta \le 1$) in buying the particular brands in our data sets. On the contrary, performance of (∟, $\delta = 1$) WTP-choice model is slightly worse than that of (△, $\delta \le 1$) WTP-choice model except for candy melt data where it performs slightly better by 0.47%.

Table 7 points out WTP-choice model with the highest L_{wtp} for each data set. It specifies WTP-choice model and the estimated parameters. More importantly, it compares the log-likelihood of the best WTP-choice model with that of Logit and mixed Logit models. We see that the best WTP-choice model is one of

Table 6 Changes and Differences in Log-Likelihoods for □, ∟ Distributed Willingness To Pays (WTPs) and $\delta = 1$

	(□, $\delta = 1$) WTP choice model			(∟, $\delta = 1$) WTP choice model		
	L_{wtp}	Change in %	Difference in %	L_{wtp}	Change in %	Difference in %
Yogurt	−1853	−0.93	−1.15	−1853	−0.93	−1.15
Ketchup	−4560	−9.83	−9.56	−4164	−0.22	−0.05
Candy melt	−11,634	−0.64	−1.20	−11,506	0.47	−0.09
Tuna	−13,157	−19.35	−18.07	−11,225	−1.82	−0.74

Table 7 Best Willingness To Pay (WTP)-Choice Model vs. Logit Models

	Best WTP-choice				
	Model	L_{wtp}	L_{sl}	L_{ml}	$(L_{wtp} - L_{ml})/\lvert L_{wtp}\rvert$ in %
Yogurt	$(\triangle, \delta \le 1)$	−1836	−1835	−1832	−0.22
Ketchup	$(\square, \delta = 1)$	−3755	−4169	−4162	10.84
Candy melt	$(\llcorner, \delta \le 1)$	−11,506	−11,498	−11,496	−0.09
Tuna	$(\triangle, \delta \le 1)$	−11,024	−11,143	−11,143	1.08

$(\square, \delta = 1)$, $(\triangle, \delta \le 1)$ or $(\llcorner, \delta \le 1)$. LL_{wtp} values for $(\square, \delta = 1)$ with ketchup data are different in Tables 6 and 7 as the former table is for estimating only (ϕ, b_1, b_2), whereas the latter is for estimating (a_1, a_2) in addition. In Table 7, with yogurt and candy melt data, mixed Logit model performs marginally (at most 0.22%) better. With the other data sets, WTP-choice model performs significantly better (as much as 10.84%). In light of these comparisons, it is safe to propose WTP-choice model as a competitive alternative to Logit models for low-price items.

6.3. Prediction Accuracy Test

We test the accuracy of sales and price predictions made with WTP-choice and Logit models. For sales prediction test, we first split each data set into two sets of equal sizes. One of the sets is referred to as estimation data while the other is test data. In the first step, we estimate the parameters for all models using the estimation data. Next, we use the estimated parameters to compute the MPE in (expected) sales with all models and the test data. Table 8 provides MPEs in % computed as $(\sum_{m=1}^{M} \lvert \sum_{n=1}^{N} [\{\hat{\rho}_n^m, \hat{\lambda}_n^m\}] - [\text{actual sales}]^m \rvert)/(\sum_{m=1}^{M} [\text{actual sales}]^m)$ for each test data set, where $\hat{\rho}_n^m$ and $\hat{\lambda}_n^m$ are predicted choice probabilities.

$(\triangle, \delta \le 1)$ WTP-choice model always predicts the sales more accurately than Logit models for $M = 2$ in Table 8. Moreover, $(\triangle, \delta \le 1)$ WTP-choice model predicts the sales better than $(\llcorner, \delta \le 1)$ and $(\square, \delta = 1)$ WTP-choice models, except for ketchup data where $(\square, \delta = 1)$ has an error of 4.23% compared with 7.86% for $(\triangle, \delta \le 1)$. For $M = 3$ products, we study consideration sets of size $L = 2$. There standard Logit model performs better with tuna data while mixed Logit and $(\square, \delta = 1)$ WTP-Choice model perform better with the other three data sets for predicting sales.

For price prediction test, a centralized revenue maximization objective (with $c = 0$) employing (standard and mixed) Logit models, WTP-choice $(\square, \delta = 1)$ model, or WTP-choice $(\triangle, \delta \le 1)$ model is used to obtain optimal prices (p^{1*}, p^{2*}). These prices are compared in Table 9 with the average of the observed prices $(\bar{p}^1, \bar{p}^2)$ in the empirical data. Optimal prices predicted (calculated) from the Logit models show higher deviations from empirical averages than the prices predicted from the WTP-choice models for yogurt and candy melt data. Logit-based prices are relatively closer to empirical prices for the ketchup and tuna data, where the price coefficient estimate $\hat{\beta}$ is relatively larger with respect to the other price coefficient estimates in the rest of data. Price predictions based on WTP-choice $(\square, \delta = 1)$ and $(\triangle, \delta \le 1)$ models are similar for all data sets except for tuna, where predictions from $(\triangle, \delta \le 1)$ are substantially better than $(\square, \delta = 1)$—an observation in line with the tuna results of Table 8. In view of our likelihood computations as well as sales and price prediction tests with various real-life data, WTP-choice models compare well with Logit models to predict the sales and optimal prices for low-price and high-purchase frequency items.

7. Concluding Remarks

This study has presented a choice model that captures a customer's WTPs and preferences. WTP-choice model contributes to the choice literature, where customers follow a simple utility satisficing rule and hence have bounded rationality. WTP-choice model

Table 8 Mean Percentage Error in Sales for Logit Models vs. Willingness To Pay (WTP)-Choice Models

	$M = 2$ products					$M = 3$ products		
	Logit models		WTP-choice models			Logit models		WTP-choice model $\square$,
	Standard	Mixed	$\llcorner, \delta \le 1$	$\square, \delta = 1$	$\triangle, \delta \le 1$	Standard	Mixed	$\delta = 1$
Yogurt	2.52	2.31	7.20	2.21	2.17	4.80	4.80	4.90
Ketchup	8.84	8.66	38.86	4.23	7.86	4.78	4.34	4.73
Candy melt	2.30	2.29	19.24	2.26	0.29	1.17	1.17	1.10
Tuna	5.06	5.06	9.84	43.65	2.71	2.84	3.51	11.12

Table 9 Comparison of Observed Prices and Optimal Price Predictions from Logit and Willingness To Pay (WTP)-Choice Models

	Averages of the observed prices		Logit models				WTP-choice models			
			Standard		Mixed		□, $\delta = 1$		△, $\delta \leq 1$	
	$\bar{p}^1$	$\bar{p}^2$	$p^{1*} - \bar{p}^1$	$p^{2*} - \bar{p}^2$	$p^{1*} - \bar{p}^1$	$p^{2*} - \bar{p}^2$	$p^{1*} - \bar{p}^1$	$p^{2*} - \bar{p}^2$	$p^{1*} - \bar{p}^1$	$p^{2*} - \bar{p}^2$
Yogurt	8.11	10.79	1.64	−1.04	11.69	9.01	2.03	−1.37	0.38	0.45
Ketchup	0.93	1.36	0.17	−0.26	0.17	−0.26	−0.31	−0.31	−0.27	−0.28
Candy melt	2.80	2.79	5.59	5.58	5.59	5.58	−1.75	0.97	0.77	0.56
Tuna	0.74	0.74	−0.19	−0.19	−0.19	−0.19	3.02	8.07	−0.26	−0.16

has several desirable properties: it explicitly captures the sequence of products considered and requires limited information; the choice probabilities are the same when prices are learnt first or last; and it does not have the IIA property.

Under competitive pricing with independent WTPs and no inventory consideration, we show that retailers are loosely coupled—equilibrium profits are coupled but prices are not. Loose coupling simplifies the computation of equilibrium and facilitates the implementation of the equilibrium prices in a competitive market. However, when the WTPs are dependent or the preferences are dependent on the prices, loose coupling fails and the retailers must consider the competitor's price while deciding on own prices. Furthermore, setting price at par with the competitor is not necessarily the equilibrium strategy even when retailers have identical products and costs. We also show that price cycles exist in competitive markets where the WTPs of the products are dependent, and provide conditions to eliminate price cycles to guarantee the existence of a price-pair equilibrium. Hence, we can analytically explain the presence of price cycles in practice.

Loose coupling of prices also fails when a single retailer selling multiple products maximizes his profit. The analysis of this centralized pricing setting interestingly turns out to be more challenging than the competitive pricing setting. An iterative algorithm CIPA is proposed to find centrally optimal prices and is shown to yield the globally optimal prices when WTPs have uniform or shifted exponential distributions. We prove that centrally optimal prices are higher than equilibrium prices. Through an example, we illustrate a counterintuitive result that a (stochastic) rise in the WTP of a product can actually reduce the optimal price of that product.

The sequential decision making structure of WTP-choice models helps us study lost sales that alter customer choices, under which loose coupling remains valid. We derive equilibrium profits and prices for the case of lost sales and observe that lost sales affect profits but not the prices. The lost sales model can be modified to study backorders; various backorder models can be envisioned and analyzed in detail but these models are not presented here to focus on the WTP-choice model.

For empirical validation, we compare the fit and prediction accuracy of standard Logit, mixed Logit, and WTP-choice models by using real-life data. Our real-life data consist of products with low price and high purchase frequency for which customers are likely to use utility satisficing (WTP-choice) rather than utility maximizing (Logit). WTP-choice models usually perform better than or on par with Logit models. WTP-choice model can also be used to estimate WTP of customers as the model is flexible and does not assume a specific distribution of WTP. In the final analysis, we are proposing WTP-choice models as alternatives that can be considered along with Logit models, but not to replace Logit models.

Willingness To Pay-choice model has a simple satisficing assumption for the customers, is designed to be versatile due to general and dependent WTP distributions, and yields simple competitive price formulas for independent WTPs. It can serve as a basis for interesting future studies such as further empirical studies to investigate distributions for and independence of WTPs and more dynamic/detailed pricing applications.

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Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix S1: Proofs.